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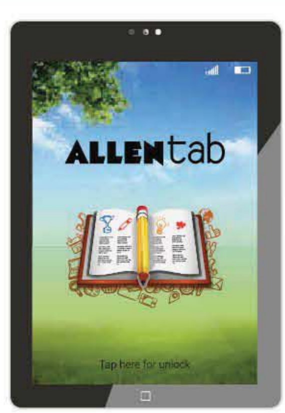

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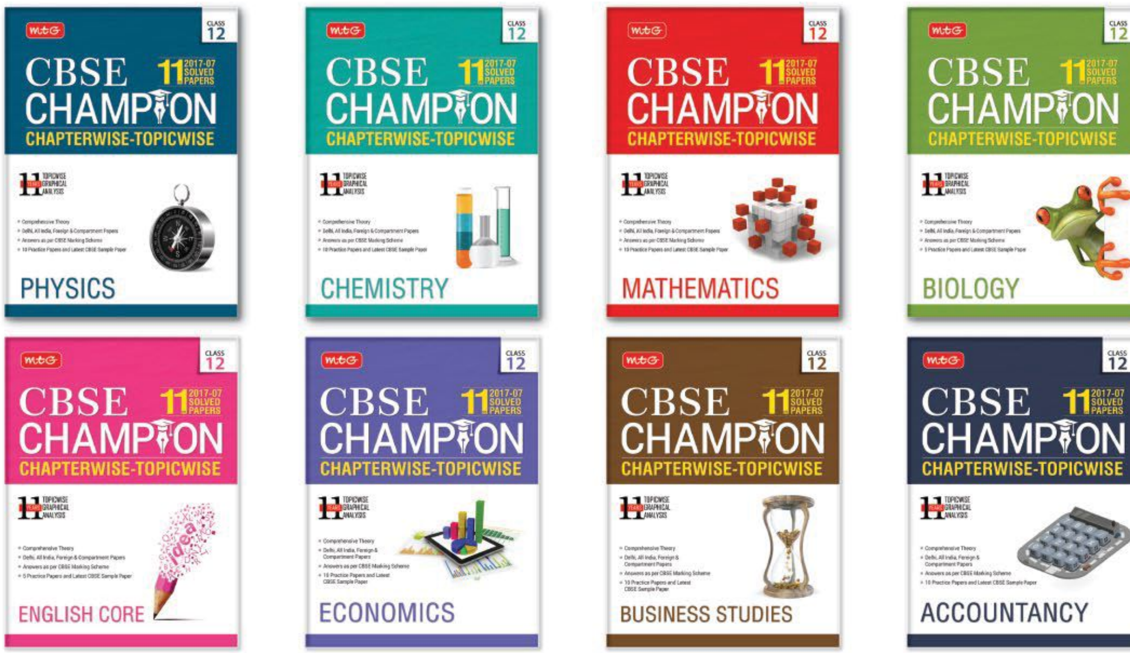

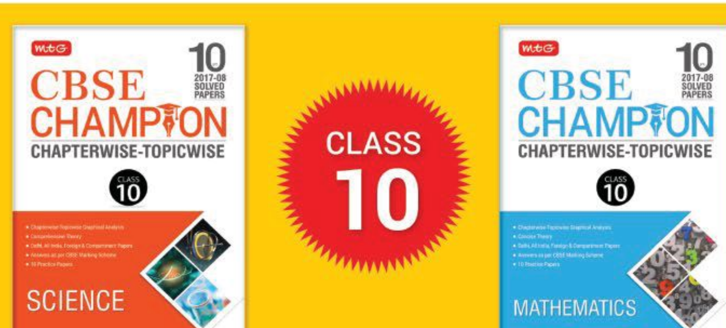

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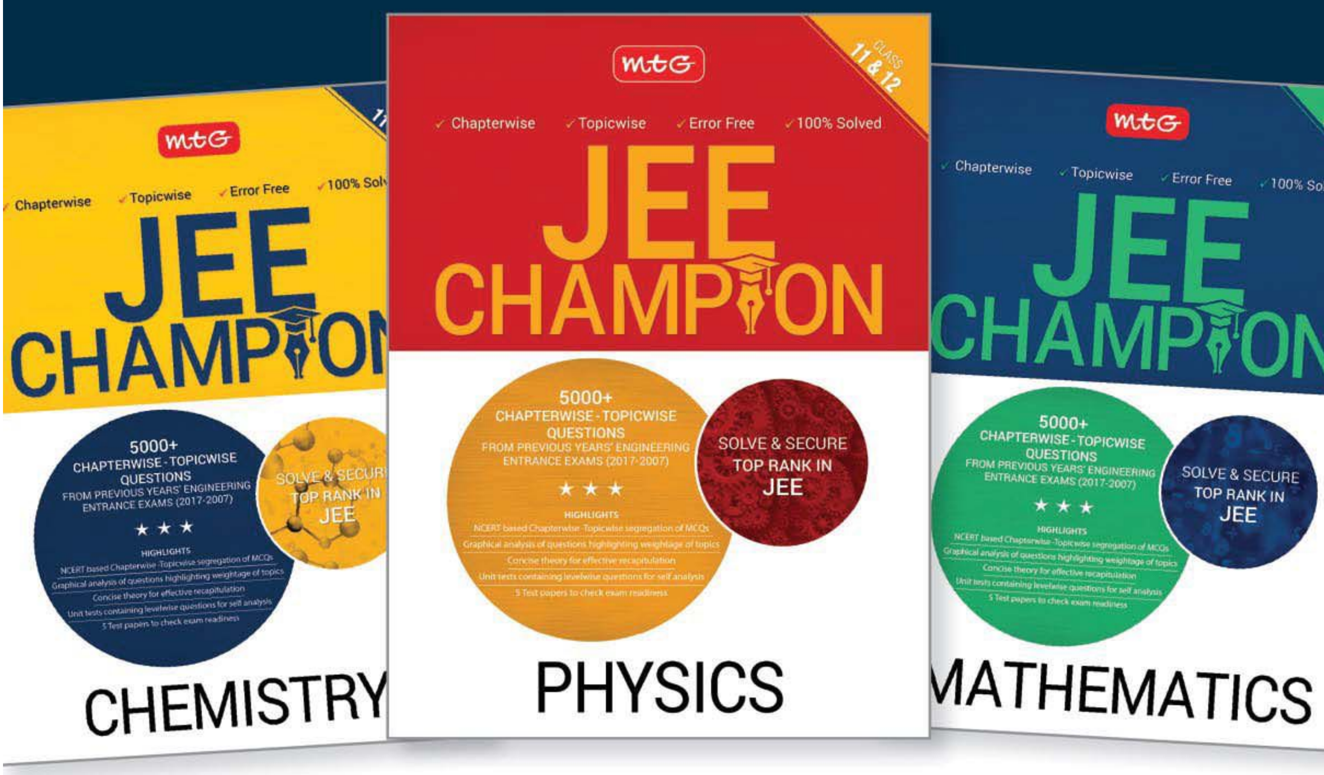

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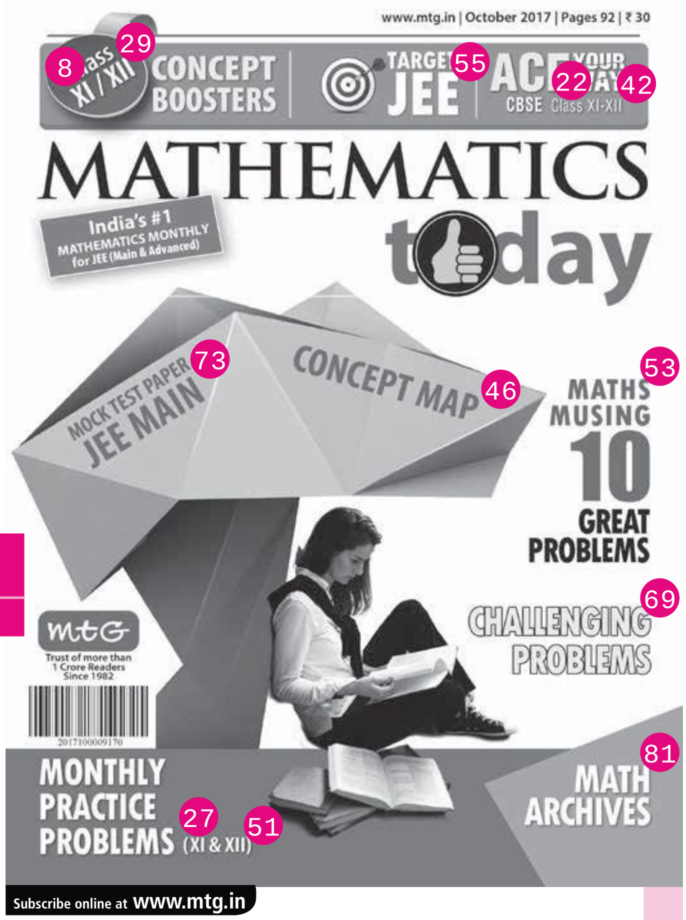

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CONCEPT BOOSTERS

Sequences and Series

Class XI

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR, B.Tech, IIT Kanpur

- A progression is a sequence whose terms follow a certain pattern *i.e.*, the terms are arranged under a definite rule.

ARITHMETIC PROGRESSION

A sequence of numbers $< t_n >$ is said to be in arithmetic progression (A.P.) when the difference $t_n - t_{n-1}$ is a constant for all $n \in N$. This constant is called the common difference of the A.P. and is usually denoted by the letter d.

If 'a' is the first term and 'd' the common difference, then an A.P. can be represented as $a, a + d, a + 2d, a + 3d, ...$

General Term of an A.P.

- n^{th} term of an A.P *i.e.*, $T_n = a + (n - 1)d$.
- p^{th} term from the end is $(n - p + 1)^{\text{th}}$ term from the beginning $= T_{(n-p+1)} = a + (n - p)d$. Also, if last term of an A.P. is l then term from end is $l - (p - 1)d$.

Sum of 'n' Terms of an A.P.

The sum of n terms of the series

$a + (a + d) + (a + 2d) + ... \{a + (n - 1)d\}$ is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = \frac{n}{2}(a+l),$$

where l = last term $= a + (n - 1)d$

ARITHMETIC MEAN

- Single A.M. between a and b : If a and b are two real numbers then single A.M. between a and b is $\frac{a+b}{2}$.
- n A.M.'s between a and b : If $A_1, A_2, A_3, ..., A_n$ are n A.M.'s between a and b, then

$$A_1 = a + d = a + \frac{b-a}{n+1}, \quad A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

$$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right)$$

Properties of A.P. : Let $a_1, a_2, a_3, ...$ are in A.P. whose common difference is d.

- For a fixed non-zero number $k \in R$
  $a_1 \pm k, a_2 \pm k, a_3 \pm k, ... ; ka_1, ka_2, ka_3, ... ;$
  $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k} ....$ will be in A.P., whose common difference will be d, kd, d/k respectively.
- The sum of terms of an A.P. equidistant from the beginning and the end is constant and is equal to sum of first and last term.
  i.e. $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$ and so on
- If number of terms of any A.P. is odd, then sum of the terms is equal to product of middle term and number of terms.
- If number of terms of any A.P. is even then A.M. of middle two terms is A.M. of first and last term.
- If the number of terms of an A.P. is odd then its middle term is A.M. of first and last term.
- If $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ are the two A.P.'s. Then $a_1 \pm b_1, a_2 \pm b_2, ... a_n \pm b_n$ are also A.P.'s with common difference $d_1 \pm d_2$, where d_1 and d_2 are the common difference of the given A.P.'s.
- Three numbers a, b, c are in A.P. iff $2b = a + c$.

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91).
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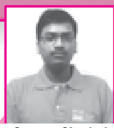
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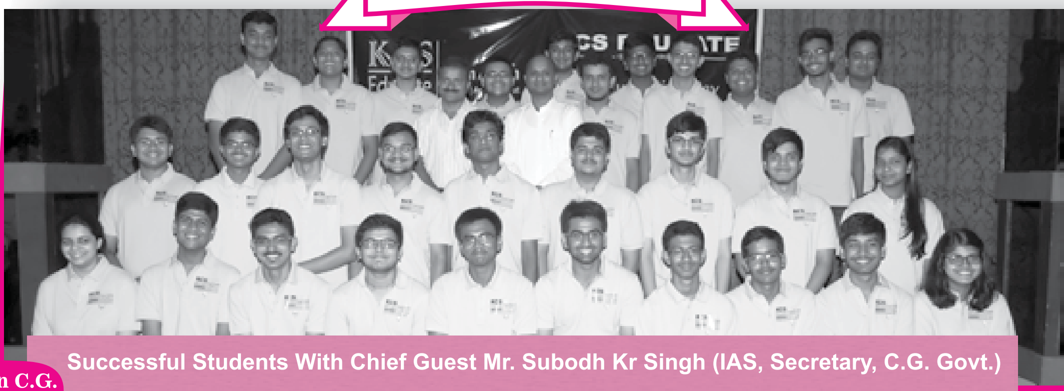

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GEOMETRIC PROGRESSION (G.P.)

A progression is called a G.P. if the ratio of its each term to its previous term is always constant. If 'a' be the first term and 'r' be the common ratio then, a, ar, ar^2, ..., ar^{n-1} is a sequence of G.P.

General Term of a G.P.

- n^{th} term of a G.P. *i.e.*, $T_n = ar^{n-1}$.

  where common ratio, $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = ...$
- If G.P. consists of 'n' terms, then p^{th} term from the end $= (n-p+1)^{th}$ term from the beginning $= ar^{n-p}$. Also, the p^{th} term from the end of a G.P. with last term l and common ratio r is $l\left(\frac{1}{r}\right)^{n-1}$.

Sum of First 'n' Terms of G.P.

Sum of first n terms of a G.P. is given by

- $S_n = \frac{a(1-r^n)}{1-r}$ and $S_n = \frac{a-lr}{1-r}$, (when $|r|<1$)
- $S_n = \frac{a(r^n-1)}{r-1}$ and $S_n = \frac{lr-a}{r-1}$, (when $|r|>1$)
- $S_n = na$, (when $r = 1$)

Sum of Infinite Terms of G.P.

- When $|r| < 1$, (or $-1 < r < 1$) ; $S_\infty = \frac{a}{1-r}$
- If $r \geq 1$, then S_∞ doesn't exist.

GEOMETRIC MEAN

- Single G.M. between a and b : If a and b are two real numbers then single G.M. between a and b $= \sqrt{ab}$.
- n G.M.'s between a and b : If $G_1, G_2, G_3, ..., G_n$ are n G.M.'s between a and b, then

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}},\ G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, ..., G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Properties of G.P.

- If all the terms of a G.P. be multiplied or divided by the same non-zero constant, then it remains a G.P., with the same common ratio.
- The reciprocal of the terms of a given G.P. form a G.P. with common ratio as reciprocal of the common ratio of the original G.P.
- If each term of a G.P. with common ratio r be raised to the same power k, the resulting sequence also forms a G.P. with common ratio r^k.
- In a finite G.P., the product of terms equidistant from the beginning and the end is always the same and is equal to the product of the first and last term. *i.e.*, if $a_1, a_2, a_3, ..., a_n$ be in G.P., Then,

  $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = a_4 a_{n-3} = ... = a_r a_{n-r+1}$
- If the terms of a given G.P. are chosen at regular intervals, then the new sequence so formed also forms a G.P.
- If $a_1, a_2, a_3, ..., a_n, ...$ is a G.P. of non-zero, non-negative terms, then $\log a_1, \log a_2, \log a_3, ..., \log a_n$, ... is an A.P. and vice-versa.
- Three non-zero numbers a, b, c are in G.P., iff $b^2 = ac$.
- If first term of a G.P. of n terms is a and last term is l, then the product of all terms of the G.P. is $(al)^{n/2}$.

HARMONIC PROGRESSION (H.P.)

- A progression is called a harmonic progression (H.P.) if the reciprocals of its terms are in A.P.

General Term of an H.P.

General term T_n of H.P. is $\frac{1}{a+(n-1)d}$

HARMONIC MEAN

If three or more numbers are in H.P., then the numbers lying between the first and last are called harmonic means (H.M.'s) between them. For example 1, 1/3, 1/5, 1/7, 1/9 are in H.P. So 1/3, 1/5 and 1/7 are three H.M.'s between 1 and 1/9.

- **Insertion of harmonic means**
  - Single H.M. between a and b $= \frac{2ab}{a+b}$
  - H.M. (H) of n non-zero numbers $a_1, a_2, a_3, ..., a_n$ is given by $\frac{1}{H} = \frac{\frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}}{n}$
  - Let a, b be two given numbers and n harmonic means $H_1, H_2, ..., H_n$ are inserted between a and b

    Now, $a, H_1, H_2, ..., H_n, b$ are in H.P.

    $\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, ..., \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

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Let D be the common difference of this A.P.

Then, $\frac{1}{b}=(n+2)^{th}$ term $=T_{n+2}$

$$\frac{1}{b}=\frac{1}{a}+(n+1)D \Rightarrow D=\frac{a-b}{(n+1)ab}$$

Properties of H.P.

- No term of H.P. can be zero.
- If H is the H.M. between a and b, then
  - $\frac{1}{H-a}+\frac{1}{H-b}=\frac{1}{a}+\frac{1}{b}$
  - $(H-2a)(H-2b)=H^2$
  - $\frac{H+a}{H-a}+\frac{H+b}{H-b}=2$

ARITHMETICO–GEOMETRIC PROGRESSION(A.G.P)

The combination of arithmetic and geometric progression is called arithmetico-geometric progression. If $a_1, a_2, a_3, ..., a_n$ is an A.P. and $b_1, b_2, b_3, ..., b_n$ is a G.P., then the sequence $a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n$ is said to be an arithmetico-geometric sequence.

General Term of A.G.P.

The general form of A.G.P is

$a,(a+d)r,(a+2d)r^2,(a+3d)r^3,.....$

$\therefore \quad T_n=[a+(n-1)d]r^{n-1}.$

Sum of A.G.P.

- Sum of n terms : The sum of n terms of an arithmetico-geometric sequence a, $(a+d)r$, $(a+2d)r^2$, $(a+3d)r^3$, ... is given by

$$S_n=\begin{cases}\frac{a}{1-r}+dr\frac{(1-r^{n-1})}{(1-r)^2}-\frac{\{a+(n-1)d\}r^n}{1-r}, & \text{when } r\neq 1\\ \frac{n}{2}[2a+(n-1)d], & \text{when } r=1\end{cases}$$

- Sum of infinite sequence :

$$S_\infty=\frac{a}{1-r}+\frac{dr}{(1-r)^2}, |r|<1.$$

Method for Finding Sum

This method is applicable for both sum of n terms and sum of infinite number of terms.

First suppose that sum of the series is S, then multiply it by common ratio of the G.P. and subtract. In this way, we shall get a G.P., whose sum can be easily obtained.

Method of Difference

If the differences of the successive terms of a series are in A.P. or G.P., we can find n^{th} term of the series by the following steps :

- Denote the n^{th} term by T_n and the sum of the series upto n terms by S_n.
- Rewrite the given series with each term shifted by one place to the right.
- By subtracting the later series from the former, find T_n.
- From T_n, S_n can be found by appropriate summation.

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- $1+3+5+.....+(2n-1)=\sum_{r=1}^{n}(2r-1)=n^2$
- $2+4+6+......+2n=\sum_{r=1}^{n}2r=n(n+1)$

Properties of Arithmetic, Geometric, Harmonic Means between Two Given Numbers

Let A, G and H be arithmetic, geometric and harmonic means of two numbers a and b.

Then, $A=\frac{a+b}{2}, G=\sqrt{ab}$ and $H=\frac{2ab}{a+b}$

- $A\geq G\geq H$

  $\because \quad A-G=\frac{a+b}{2}-\sqrt{ab}=\frac{(\sqrt{a}-\sqrt{b})^2}{2}\geq 0$

  $\Rightarrow \quad A\geq G \qquad ...(i)$

  $G-H=\sqrt{ab}-\frac{2ab}{a+b}=\sqrt{ab}\left(\frac{a+b-2\sqrt{ab}}{a+b}\right)$

  $=\frac{\sqrt{ab}}{a+b}(\sqrt{a}-\sqrt{b})^2\geq 0$

  $\Rightarrow \quad G\geq H \qquad ...(ii)$

  From (i) and (ii), we get $A\geq G\geq H$

  Note that the equality holds only when $a=b$.
- A, G, H form a G.P., *i.e.*, $G^2=AH$
- The equation having a and b as its roots is

$$x^2 - (a+b)x + ab = 0$$
$$\Rightarrow \quad x^2 - 2Ax + G^2 = 0$$
$$\left[\because \quad A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}\right]$$

The roots a, b are given by $A \pm \sqrt{A^2 - G^2}$.

The equation having a, b, c as its roots is

$$x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc = 0$$

Here $A = \dfrac{a+b+c}{3}, G = (abc)^{1/3}$ and $\dfrac{1}{H} = \dfrac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}$

$$\Rightarrow \quad a+b+c = 3A, abc = G^3 \text{ and } \frac{3G^3}{H} = ab+bc+ca$$
$$\Rightarrow \quad x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0$$

RELATION BETWEEN A.P., G.P. AND H.P.

- If A, G, H be A.M., G.M., H.M. between a and b, then $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \begin{cases} A \text{ when } n = 0 \\ G \text{ when } n = -1/2 \\ H \text{ when } n = -1 \end{cases}$
- If A_1, A_2 be two A.M.'s; G_1, G_2 be two G.M.'s and H_1, H_2 be two H.M.'s between two numbers a and b, then $\dfrac{G_1G_2}{H_1H_2} = \dfrac{A_1+A_2}{H_1+H_2}$
- Recognization of A.P., G.P., H.P. : If a, b, c are three successive terms of a sequence.

  If $\dfrac{a-b}{b-c} = \dfrac{a}{a}$, then a, b, c are in A.P.

  If, $\dfrac{a-b}{b-c} = \dfrac{a}{b}$, then a, b, c are in G.P.

  If, $\dfrac{a-b}{b-c} = \dfrac{a}{c}$, then a, b, c are in H.P.
- If number of terms of any A.P./G.P./H.P. is odd, then A.M./G.M./H.M. of first and last terms is middle term of series.
- If number of terms of any A.P./G.P./H.P. is even, then A.M./G.M./H.M. of middle two terms is A.M./G.M./H.M. of first and last terms respectively.
- If p^{th}, q^{th} and r^{th} terms of a G.P. are in G.P. Then p, q, r are in A.P.
- If a, b, c are in A.P. as well as in G.P. then $a = b = c$.
- If a, b, c are in A.P., then x^a, x^b, x^c will be in G.P. $(x \neq \pm 1)$

IMPORTANT POINTS

- If T_k and T_p of any A.P. are given, then formula for obtaining T_n is $\dfrac{T_n - T_k}{n-k} = \dfrac{T_p - T_k}{p-k}$.
- If $pT_p = qT_q$ of an A.P., then $T_{p+q} = 0$.
- If p^{th} term of an A.P. is q and the q^{th} term is p, then $T_{p+q} = 0$ and $T_n = p + q - n$.
- If the p^{th} term of an A.P. is $\dfrac{1}{q}$ and the q^{th} term is $\dfrac{1}{p}$ then $T_{pq} = 1$ and $S_{pq} = \dfrac{1}{2}(pq+1)$
- The common difference of an A.P is given by $d = S_2 - 2S_1$ where S_2 is the sum of first two terms and S_1 is the sum of first term or the first term.
- If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$, where S_{n-1} is sum of $(n-1)$ terms of A.P.
- If for an A.P. sum of p terms is q and sum of q terms is p, then sum of $(p+q)$ terms is $\{-(p+q)\}$.
- If for an A.P., sum of p terms is equal to sum of q terms, then sum of $(p+q)$ terms is zero.
- Sum of n A.M.'s between a and b is equal to n times the single A.M. between a and b.

  i.e., $A_1 + A_2 + A_3 + \ldots\ldots + A_n = n\left(\dfrac{a+b}{2}\right)$
- If A_1 and A_2 are two A.M.'s between two numbers a and b, then $A_1 = \dfrac{1}{3}(2a+b), A_2 = \dfrac{1}{3}(a+2b)$.
- Between two numbers, $\dfrac{\text{Sum of } m \text{ A.M.'s}}{\text{Sum of } n \text{ A.M.'s}} = \dfrac{m}{n}$
- If T_k and T_p of any G.P. are given, then formula for obtaining T_n is $\left(\dfrac{T_n}{T_k}\right)^{\frac{1}{n-k}} = \left(\dfrac{T_p}{T_k}\right)^{\frac{1}{p-k}}$.
- If a, b, c are in G.P. then

  $\Rightarrow \dfrac{b}{a} = \dfrac{c}{b} \Rightarrow \dfrac{a+b}{a-b} = \dfrac{b+c}{b-c}$

  or $\dfrac{a-b}{b-c} = \dfrac{a}{b}$ or $\dfrac{a+b}{b+c} = \dfrac{a}{b}$
- Let the first term of a G.P be positive, then if $r > 1$, then it is an increasing G.P., but if r is positive and less than 1, *i.e.* $0 < r < 1$, then it is a decreasing G.P.
- Let the first term of a G.P. be negative, then if $r > 1$, then it is a decreasing G.P., but if $0 < r < 1$, then it is an increasing G.P.

- Product of n G.M.'s between a and b is equal to n^{th} power of single geometric mean between a and b, *i.e.*, $G_1 G_2 G_3......G_n = (\sqrt{ab})^n$
- G.M. of $a_1\, a_2\, a_3 \ldots a_n$ is $(a_1\, a_2\, a_3 \ldots a_n)^{1/n}$.
- If G_1 and G_2 are two G.M.'s between two numbers a and b is $G_1 = (a^2b)^{1/3}, G_2 = (ab^2)^{1/3}$.
- If H_1 and H_2 are two H.M.'s between a and b, then $H_1 = \frac{3ab}{a+2b}$ and $H_2 = \frac{3ab}{2a+b}$.

PROBLEMS

Single Correct Answer Type

1. The number of terms in the series
101 + 99 + 97 + ... + 47 is
(a) 25 (b) 28 (c) 30 (d) 20

2. If $\tan n\theta = \tan m\theta$, then the different values of θ will be in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

3. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
(a) 3000 (b) 3050
(c) 4050 (d) none of these

4. If the sum of n terms of an A.P. is $nA + n^2B$, where A, B are constants, then its common difference will be
(a) $A - B$ (b) $A + B$
(c) $2A$ (d) $2B$

5. If the 9th term of an A.P. is 35 and 19th is 75, then its 20th terms will be
(a) 78 (b) 79 (c) 80 (d) 81

6. The 9th term of the series $27 + 9 + 5\frac{2}{5} + 3\frac{6}{7} + ...$ will be
(a) $1\frac{10}{17}$ (b) $\frac{10}{17}$ (c) $\frac{16}{27}$ (d) $\frac{17}{27}$

7. If a, b, c are in A.P., then $\frac{(a-c)^2}{(b^2-ac)} =$
(a) 1 (b) 2 (c) 3 (d) 4

8. If the p^{th}, q^{th} and r^{th} term of an arithmetic sequence are a , b and c respectively, then the value of $[a(q - r) + b(r - p) + c(p - q)] =$
(a) 1 (b) -1 (c) 0 (d) 1/2

9. If p times the p^{th} term of an A.P. is equal to q times the q^{th} term of an A.P., then $(p + q)^{th}$ term is
(a) 0 (b) 1 (c) 2 (d) 3

10. The sums of n terms of two arithmetic series are in the ratio $2n + 3 : 6n + 5$, then the ratio of their 13th terms is
(a) 53 : 155 (b) 27 : 77
(c) 29 : 83 (d) 31 : 89

11. Let T_r be the r^{th} term of an A.P. for $r = 1, 2, 3, ...$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals
(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

12. If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P. then x equals
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

13. If the ratio of the sum of n terms of two A.P.'s be $(7n+1):(4n+27)$, then the ratio of their 11th terms will be
(a) 2 : 3 (b) 3 : 4 (c) 4 : 3 (d) 5 : 6

14. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5°, then the number of sides is
(a) 8 (b) 10 (c) 9 (d) 6

15. If the p^{th} term of an A.P. be $1/q$ and q^{th} term be $1/p$, then the sum of its pq^{th} terms will be
(a) $\frac{pq-1}{2}$ (b) $\frac{1-pq}{2}$
(c) $\frac{pq+1}{2}$ (d) $-\frac{pq+1}{2}$

16. If the first, second and last terms of an A.P. be a, b, $2a$ respectively, then its sum will be
(a) $\frac{ab}{b-a}$ (b) $\frac{ab}{2(b-a)}$
(c) $\frac{3ab}{2(b-a)}$ (d) $\frac{3ab}{4(b-a)}$

17. If $a_1, a_2, ..., a_n$ are in A.P. with common difference d, then the sum of the following series is
$\sin d(\operatorname{cosec} a_1 \cdot \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \cdot \operatorname{cosec} a_3 + ... + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n)$
(a) $\sec a_1 - \sec a_n$ (b) $\cot a_1 - \cot a_n$
(c) $\tan a_1 - \tan a_n$ (d) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

18. If the sum of the series 2 + 5 + 8 + 11 + ... is 60100, then the number of terms are
(a) 100 (b) 200 (c) 150 (d) 250

19. The sum of all natural numbers between 1 and 100 which are multiples of 3 is
(a) 1680 (b) 1683 (c) 1681 (d) 1682

20. If the sum of n terms of an A.P. is $2n^2 + 5n$, then the n^{th} term will be
(a) $4n + 3$ (b) $4n + 5$ (c) $4n + 6$ (d) $4n + 7$

21. The n^{th} term of an A.P. is $3n - 1$.Choose from the following the sum of its first five terms
(a) 14 (b) 35 (c) 80 (d) 40

22. The sum of the numbers between 100 and 1000 which is divisible by 9 will be
(a) 55350 (b) 57228 (c) 97015 (d) 62140

23. The ratio of sum of m and n terms of an A.P. is $m^2 : n^2$, then the ratio of m^{th} and n^{th} term will be
(a) $\frac{m-1}{n-1}$ (b) $\frac{n-1}{m-1}$ (c) $\frac{2m-1}{2n-1}$ (d) $\frac{2n-1}{2m-1}$

24. The solution of
$\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + ... + \log_{\sqrt[16]{3}} x = 36$ is
(a) $x = 3$ (b) $x = 4\sqrt{3}$
(c) $x = 9$ (d) $x = \sqrt{3}$

25. A series whose n^{th} term is $\left(\frac{n}{x}\right) + y$, then the sum of r terms will be
(a) $\left\{\frac{r(r+1)}{2x}\right\} + ry$ (b) $\left\{\frac{r(r-1)}{2x}\right\}$
(c) $\left\{\frac{r(r-1)}{2x}\right\} - ry$ (d) $\left\{\frac{r(r+1)}{2y}\right\} - rx$

26. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is
(a) 2489 (b) 4735 (c) 2317 (d) 2632

27. If $a_1, a_2, ..., a_{n+1}$ are in A.P., then
$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_na_{n+1}}$ is
(a) $\frac{n-1}{a_1a_{n+1}}$ (b) $\frac{1}{a_1a_{n+1}}$
(c) $\frac{n+1}{a_1a_{n+1}}$ (d) $\frac{n}{a_1a_{n+1}}$

28. Let the sequence $a_1, a_2, a_3, ..., a_{2n}$ form an A.P. Then $a_1^2 - a_2^2 + a_3^3 - .... + a_{2n-1}^2 - a_{2n}^2 =$
(a) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (b) $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$
(c) $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$ (d) None of these

29. The first term of an A.P. of consecutive integers is $p^2 + 1$. The sum of $(2p + 1)$ terms of this series can be expressed as
(a) $(p + 1)^2$ (b) $(p + 1)^3$
(c) $(2p + 1)(p + 1)^2$ (d) $p^3 + (p + 1)^3$

30. The number of terms of the A.P. 3,7,11,15...to be taken so that the sum is 406 is
(a) 5 (b) 10 (c) 12 (d) 14

31. Three numbers are in A.P. such that their sum is 18 and sum of their squares is 158. The greatest number among them is
(a) 10 (b) 11
(c) 12 (d) None of these

32. If $\frac{3+5+7+..... \text{ to } n \text{ terms}}{5+8+11+.... \text{ to } 10 \text{ terms}} = 7$, then the value of n is
(a) 35 (b) 36 (c) 37 (d) 40

33. If A_1, A_2 be two arithmetic means between 1/3 and $\frac{1}{24}$, then their values are
(a) $\frac{7}{72}, \frac{5}{36}$ (b) $\frac{17}{72}, \frac{5}{36}$
(c) $\frac{7}{36}, \frac{5}{72}$ (d) $\frac{5}{72}, \frac{17}{72}$

34. A number is the reciprocal of the other. If the arithmetic mean of the two numbers be $\frac{13}{12}$, then the numbers are
(a) $\frac{1}{4}, \frac{4}{1}$ (b) $\frac{3}{4}, \frac{4}{3}$ (c) $\frac{2}{5}, \frac{5}{2}$ (d) $\frac{3}{2}, \frac{2}{3}$

35. If A be an arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then
(a) $S = nA$ (b) $A = nS$
(c) $A = S$ (d) None of these

36. If $f(x+y, x-y) = xy$, then the arithmetic mean of $f(x, y)$ and $f(y, x)$ is
(a) x (b) y (c) 0 (d) 1

37. If the sides of a right angled traingle are in A.P., then the sides are proportional to
(a) 1 : 2 : 3 (b) 2 : 3 : 4
(c) 3 : 4 : 5 (d) 4 : 5 : 6

38. If the sum of three numbers of a arithmetic sequence is 15 and the sum of their squares is 83, then the numbers are
(a) 4, 5, 6 (b) 3, 5, 7
(c) 1, 5, 9 (d) 2, 5, 8

39. The four arithmetic means between 3 and 23 are
(a) 5, 9, 11, 13 (b) 7, 11, 15, 19
(c) 5, 11, 15, 22 (d) 7, 15, 19, 21

40. If $\frac{1}{p+q}, \frac{1}{r+p}, \frac{1}{q+r}$ are in A.P., then
(a) p, q, r are in A.P. (b) p^2, q^2, r^2 are in A.P.
(c) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P.
(d) None of these

41. If $1, \log_y x, \log_z y, -15\log_x z$ are in A.P., then
(a) $z^3 = x$ (b) $x = y^{-1}$
(c) $z^{-3} = y$ (d) $x = y^{-1} = z^3$

Multiple Correct Answer Type

42. If in a ΔABC, a, b, c are in A.P. then it is necessary that
(a) $\frac{2}{3} < \frac{b}{c} < 2$ (b) $\frac{1}{3} < \frac{b}{c} < \frac{2}{3}$
(c) $\frac{2}{3} < \frac{b}{a} < 2$ (d) $\frac{1}{3} < \frac{b}{a} < \frac{2}{3}$

43. Let $S_1, S_2, ..., S_n$ be the sums of geometric series whose 1st terms are 1, 2, 3, ..., n and common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n+1}$ respectively. Then
(a) $S_1 + S_2 + ... + S_n = \frac{n(n+3)}{2}$
(b) $S_1 \cdot S_2 \cdot S_3 ..... S_n = \lfloor n+1$
(c) $\frac{1}{S_1S_2} + \frac{1}{S_2S_3} + ... \frac{1}{S_{n-1}S_n} = \frac{n-1}{2(n+1)}$
(d) $S_1^2 \cdot S_2^3 \cdot S_3^4 .... S_n^{n+1} = \frac{1024}{3}$

44. If $\sum_{r=1}^{n} r(r+1) = \frac{(n+a)(n+b)(n+c)}{3}$, where $a < b < c$, then
(a) $2b = c$ (b) $a^3 - 8b^3 + c^3 = 8abc$
(c) c is prime number (d) $(a + b)^2 = 0$

45. Let $a_n = \underset{n \text{ times}}{(111, ...1)}$, then
(a) a_{912} is not prime (b) a_{951} is not prime
(c) a_{480} is not prime (d) a_{91} is not prime

46. If a, b, c are in H.P., then
(a) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in H.P.
(b) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.
(c) $\frac{1}{a} + \frac{1}{b+c}, \frac{1}{b} + \frac{1}{c+a}, \frac{1}{c} + \frac{1}{a+b}$ are in H.P.
(d) $\frac{bc}{b+c}, \frac{ca}{c+a}, \frac{ab}{a+b}$ are in H.P.

47. If n be odd or even, then the sum of n terms of the series 1 – 2 + 3 – 4 + 5 – 6 + ... will be
(a) $-\frac{n}{2}$ (b) $\frac{n-1}{2}$
(c) $\frac{n+1}{2}$ (d) $\frac{2n+1}{2}$

Comprehension Type

Paragraph for Q. No. 48 to 52

Let $A_1, A_2, A_3, ..., A_m$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, ..., G_n$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025 × 171.

48. The value of n is
(a) 7 (b) 9
(c) 11 (d) none of these

49. The value of m is
(a) 340 (b) 342 (c) 344 (d) 346

50. The value of $G_1 + G_2 + G_3 + ... + G_n$ is
(a) 1022 (b) 2044
(c) 512 (d) none of these

51. The common difference of the progression $A_1, A_3, A_5, ..., A_{m-1}$ is
(a) 6 (b) 3 (c) 2 (d) 1

52. The numbers $2A_{171}, G_5^2 + 1, 2A_{172}$ are in
(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

Paragraph for Q. No. 53 to 55

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15 and where D and d are the common differences such that $D - d = 1$. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers respectively and $d > 0$, in the two sets

53. Value of p is
(a) 100 (b) 120 (c) 105 (d) 110

54. Value of q is
(a) 100 (b) 120 (c) 105 (d) 110

55. Value of $D + d$ is
(a) 1 (b) 2 (c) 3 (d) 4

Matrix–Match Type

56. Match the following.

	Column-I		Column-II
(A)	The sequence a, b, 10, c, d is an arithmetic progression. Then the value of $a + b + c + d$ is	(p)	52
(B)	The sides of right triangle form a three term geometric sequence. The shortest side has length 2. The length of the hypotenuse is of the form $\sqrt{a} + \sqrt{b}$ where a, b $\in N$, then $a^2 + b^2$ equals	(q)	20
(C)	The sum of first three consecutive numbers of an infinite G.P. is 70, if the two extremes be multipled each by 4, and the mean by 5, the products are in A.P. The first term of the G.P. is	(r)	26
(D)	The diagonals of a parallelogram have a measure of 4 and 6 metres. They cut off forming an angle of 60°. If the perimeter of the parallelogram is $2(\sqrt{p}+\sqrt{q})$ where $p, q \in N$ then $(p + q)$ equals	(s)	40

57. Match the following.

	Column-I		Column-II
(A)	Suppose that $F(n+1)=\dfrac{2F(n)+1}{2}$ for n = 1, 2, 3, ... and $F(1) = 2$. Then $F(101)$ equals	(p)	42
(B)	If a_1, a_2, a_3, ..., a_{21} are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$ then the value of $\sum_{i=1}^{21} a_i$ is	(q)	1620
(C)	10^{th} term of the sequence $S = 1 + 5 + 13 + 29 + ...$, is	(r)	52
(D)	The sum of all two digit numbers which are not divisible by 2 or 3 is	(s)	2045

58. Match the following.

	Column-I		Column-II
(A)	The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$, then product of the two number is	(p)	$\dfrac{240}{77}$
(B)	The sum of the series $\dfrac{5}{1^2.4^2}+\dfrac{11}{4^2.7^2}+\dfrac{17}{7^2.10^2}+...$ is	(q)	32
(C)	If the first two terms of a Harmonic Progression be $\dfrac{1}{2}$ and $\dfrac{1}{3}$ then the Harmonic Mean of the first four terms is	(r)	1/3
(D)	Geometric mean of 4 and 9	(s)	6

Integer Answer Type

59. If the sum to infinity of a decreasing G.P. with the common ratio x is 6 k such that $|x| < 1$; $x \neq 0$. The ratio of the fourth term to the second term is 1/16 and the ratio of third term to the square of the second term is 1/9. Find the value of k.

60. The sum of the terms of an infinitely decreasing G.P. is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ on the interval [–4, 3] and the difference between the first and second terms is $f'(0)$. Then the value of 3 r (where r is common ratio) is

SOLUTIONS

1. (b) : First term $a = 101$, common difference $d = -2$ and last term $l = 47$

$\therefore\ T_l = a+(n-1)d \Rightarrow 47 = 101+(n-1)(-2) \Rightarrow n = 28$

2. (a) : We have $\tan n\theta = \tan m\theta \Rightarrow n\theta = N\pi + (m\theta)$

$\Rightarrow \theta = \dfrac{N\pi}{n-m}$, putting $N = 1, 2, 3, ...,$ we get

$\dfrac{\pi}{n-m}, \dfrac{2\pi}{n-m}, \dfrac{3\pi}{n-m}, ...$

$\therefore$ Common difference $(d) = \dfrac{\pi}{n-m}$

So, different values of θ will be in A.P.

3. (b) : The sum of integers from 1 to 100 that are divisible by 2 or 5 = sum of series divisible by 2 + sum of series divisible by 5 – sum of series divisible by 2 and 5.

$= (2 + 4 + 6 + ... + 100) + (5 + 10 + 15 .... + 100) - (10 + 20 + 30 + ... + 100)$

$$= \frac{50}{2}\{2\times2+(50-1)2\}+\frac{20}{2}\{2\times5+(20-1)5\} - \frac{10}{2}\{10\times2+(10-1)10\}$$

$= 2550 + 1050 - 550 = 3050$

4. (d) : Given that $S_n = nA + n^2B$

Putting n = 1, 2, 3, ..., we get

$S_1 = A+B, S_2 = 2A+4B, S_3 = 3A+9B$ and so on

$\therefore\ T_1 = S_1 = A+B, T_2 = S_2 - S_1 = A+3B,$

Hence, common difference $(d) = 2B$.

5. (b) : $T_9 = a+8d = 35$ and $T_{19} = a+18d = 75$
On solving we get $d = 4$ and $a = 3$
Hence 20th term of an A.P. is $a+19d = 3+19\times 4 = 79$.

6. (a) : Given series $27+9+5\frac{2}{5}+3\frac{6}{7}+......$

$$= 27+\frac{27}{3}+\frac{27}{5}+\frac{27}{7}+.....+\frac{27}{2n-1}+......$$

$$\therefore \quad T_n = \frac{27}{2n-1} \Rightarrow T_9 = \frac{27}{(2\times 9)-1} = \frac{27}{17} = 1\frac{10}{17}$$

7. (d) : If a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

$$\text{So, } \frac{(a-c)^2}{(b^2-ac)} = \frac{(a-c)^2}{\left\{\left(\frac{a+c}{2}\right)^2 - ac\right\}}$$

$$= \frac{(a-c)^2 4}{[a^2+c^2+2ac-4ac]} = \frac{4(a-c)^2}{(a-c)^2} = 4$$

8. (c) **9. (a)**

10. (a) : We have $\frac{S_{n_1}}{S_{n_2}} = \frac{2n+3}{6n+5}$

$$\Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{2n+3}{6n+5}$$

Put $n = 25$ then $\frac{a_1+12d_1}{a_2+12d_2} = \frac{2(25)+3}{6(25)+5} \Rightarrow \frac{T_{13}}{T_{13_2}} = \frac{53}{155}$

11. (c)

12. (b) : The given numbers are in A.P.

$$\therefore \quad 2\log_9(3^{1-x}+2) = \log_3(4\cdot 3^x - 1)+1$$

$$\Rightarrow \quad 2\log_{3^2}(3^{1-x}+2) = \log_3(4\cdot 3^x-1)+\log_3 3$$

$$\Rightarrow \quad \frac{2}{2}\log_3(3^{1-x}+2) = \log_3[3(4\cdot 3^x-1)]$$

$$\Rightarrow \quad 3^{1-x}+2 = 3(4\cdot 3^x-1)$$

$$\Rightarrow \quad \frac{3}{y}+2 = 12y-3, \text{ where } y = 3^x$$

$$\Rightarrow \quad 12y^2-5y-3 = 0$$

$$y = \frac{-1}{3} \text{ or } \frac{3}{4} \Rightarrow 3^x = \frac{-1}{3} \text{ or } 3^x = \frac{3}{4}$$

$$x = \log_3(3/4) \Rightarrow x = 1-\log_3 4 .$$

13. (c)

14. (c) : Let the number of sides of the polygon be n
Then, the sum of interior angles of the polygon

$$= (2n-4)\frac{\pi}{2} = (n-2)\pi$$

Since the angles are in A.P. Also, $a = 120^o$, $d = 5$,

$$\therefore \quad S_n = \frac{n}{2}[2\times 120+(n-1)5] = (n-2)180$$

$$\Rightarrow n^2-25n+144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$$

But $n = 16$ gives $T_{16} = a+15d = 120^o+15\cdot 5^o = 195^o$, which is impossible as interior angle cannot be greater than 180°. Hence $n = 9$.

15. (c)

16. (c) : We have first term $A = a$ ...(i)
Second term $A + d = b$ ...(ii)
and last term $l = 2a$ ...(iii)

From (i), (ii) and (iii), $d = (b-a)$ and $n = \frac{b}{b-a}$

Then, sum $(S) = \frac{n}{2}[a+l] = \frac{b}{2(b-a)}[a+2a] = \frac{3ab}{2(b-a)}$

17. (b) : As given $d = a_2-a_1 = a_3-a_2 = ... = a_n-a_{n-1}$

$$\therefore \quad \sin d\{\operatorname{cosec} a_1 \operatorname{cosec} a_2+.....+\operatorname{cosec} a_{n-1}\operatorname{cosec} a_n\}$$

$$= \frac{\sin(a_2-a_1)}{\sin a_1\cdot \sin a_2}+......+\frac{\sin(a_n-a_{n-1})}{\sin a_{n-1}\sin a_n}$$

$$= (\cot a_1-\cot a_2)+(\cot a_2-\cot a_3)+....+(\cot a_{n-1}-\cot a_n)$$

$$= \cot a_1-\cot a_n$$

18. (b) : Given, Series, 2 + 5 + 8 + 11 + ... where $a = 2$, $d = 3$ and let number of terms is n

$$\therefore \quad \text{Sum of A.P.} = \frac{n}{2}\{2a+(n-1)d\}$$

$$\Rightarrow 60100 = \frac{n}{2}\{2\times 2+(n-1)3\} \Rightarrow 120200 = n(3n+1)$$

$$\Rightarrow 3n^2+n-120200 = 0 \Rightarrow (n-200)(3n+601) = 0$$

Hence $n = 200$.

19. (b) **20. (a)**

21. (d) : Here $T_n = 3n - 1$, putting $n = 1, 2, 3, 4, 5$ we get first five terms, 2, 5, 8, 11, 14
Hence sum is 2 + 5 + 8 + 11 + 14 = 40 .

22. (a) : Series 108 + 117 + ... + 999 is an A.P. where $a = 108$, common difference $d = 9$,

$$n = \frac{999}{9}-\frac{99}{9} = 111-11 = 100$$

Hence, required sum $= \frac{100}{2}(108+999)$
$= 50 \times 1107 = 55350.$

23. (c) **24. (d)**

25. (a) : On putting n = 1, 2, 3, ….., we get

First term of the series $a = \frac{1}{x} + y,$

Second term $= \frac{2}{x} + y$ and $d = \left(\frac{2}{x} + y\right) - \left(\frac{1}{x} + y\right) = \frac{1}{x}$

$\therefore$ Sum of r terms of the series

$$= \frac{r}{2}\left[2\left(\frac{1}{x} + y\right) + (r-1)\frac{1}{x}\right] = \frac{r}{2}\left[\frac{2}{x} + 2y + \frac{r}{x} - \frac{1}{x}\right]$$

$$= \frac{r^2 - r + 2r}{2x} + ry = \left[\frac{r(r+1)}{2x} + ry\right]$$

26. (d) : Let S = 1 + 2 + 3 + ….. + 100

$$= \frac{100}{2}(1+100) = 50(101) = 5050$$

Let $S_1 = 3 + 6 + 9 + 12 + ... + 99$

$$= 3(1+2+3+4+....+33) = 3 \cdot \frac{33}{2}(1+33) = 99 \times 17 = 1683$$

Let $S_2 = 5 + 10 + 15 + ... + 100$

$$= 5(1+2+3+........+20) = 5.\frac{20}{2}(1+20) = 50 \times 21 = 1050$$

Let $S_3 = 15 + 30 + 45 + ... + 90$

$$= 15(1+2+3+........+6) = 15.\frac{6}{2}(1+6) = 45 \times 7 = 315$$

$\therefore$ Required sum = $S - S_1 - S_2 + S_3$
= 5050 – 1683 – 1050 + 315 = 2632.

27. (d)

28. (a) : Since $a_1, a_2, a_3, ..., a_{2n}$ form an A.P.

$\therefore \quad a_2 - a_1 = a_4 - a_3 = ... = a_{2n} - a_{2n-1} = d$

Here $a_1^2 - a_2^2 + a_3^2 - a_4^2 + ... + a_{2n-1}^2 - a_{2n}^2$

$$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + ... \\ ...... + (a_{2n-1} - a_{2n})(a_{2n-1} + a_{2n})$$

$$= -d(a_1 + a_2 + ... + a_{2n}) = -d\left\{\frac{2n}{2}(a_1 + a_{2n})\right\}$$

Also, $2n = a_1 + (2n-1)d \Rightarrow d = \frac{a_{2n} - a_1}{2n-1}$

Sum is $= \frac{n(a_1 - a_{2n}).(a_1 + a_{2n})}{2n-1} = \frac{n}{2n-1}(a_1^2 - a_{2n}^2)$

29. (d) : $S_{2p+1} = \frac{2p+1}{2}\{2(p^2+1) + (2p+1-1)1\}$

$$= \left(\frac{2p+1}{2}\right)(2p^2 + 2p + 2) = (2p+1)(p^2 + p + 1)$$

$$= p^3 + (p+1)^3$$

30. (d) : $S = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 406 = \frac{n}{2}[6 + (n-1)4] \Rightarrow 812 = n[6 + 4n - 4]$

$\Rightarrow \quad 812 = 2n + 4n^2 \Rightarrow 406 = 2n^2 + n$

$\Rightarrow \quad 2n^2 + n - 406 = 0$

$\Rightarrow \quad n = \frac{-1 \pm \sqrt{1 + 4 \cdot 2 \cdot 406}}{2 \cdot 2} = \frac{-1 \pm \sqrt{3249}}{4} = \frac{-1 \pm 57}{4}$

$\therefore \quad n = \frac{-1 + 57}{4} = 14$

31. (b) **32. (a)**

33. (b) : Here, $\frac{1}{3}, A_1, A_2, \frac{1}{24}$ will be in A.P.,

then $A_1 - \frac{1}{3} = \frac{1}{24} - A_2 \Rightarrow A_1 + A_2 = \frac{3}{8}$ ...(i)

Now, A_1 is a arithmetic mean of $\frac{1}{3}$ and A_2, we have

$2A_1 = \frac{1}{3} + A_2 \Rightarrow 2A_1 - A_2 = \frac{1}{3}$ ...(ii)

From (i) and (ii), we get, $A_1 = \frac{17}{72}$ and $A_2 = \frac{5}{36}$

34. (d) : Suppose the required numbers are a and b

Therefore according to the conditions, a = 1/6

and $\frac{a+b}{2} = \frac{13}{12} \Rightarrow a + b = \frac{13}{6}$

$\Rightarrow \quad a + \frac{1}{a} = \frac{13}{6} \Rightarrow 6a^2 - 13a + 6 = 0$

$\Rightarrow \quad \left(a - \frac{3}{2}\right)\left(a - \frac{2}{3}\right) = 0 \Rightarrow a = \frac{3}{2}$ and $b = \frac{2}{3}$

or $a = \frac{2}{3}$ and $b = \frac{3}{2}$.

35. (a)

36. (c) : Let $x + y = u,\ x - y = v$

$\Rightarrow \quad x = \frac{u+v}{2}, y = \frac{u-v}{2}, \therefore f(u, v) = \left(\frac{u+v}{2}\right).\left(\frac{u-v}{2}\right)$

Now $\frac{f(x, y) + f(y, x)}{2}$

$$= \frac{\left(\frac{x+y}{2}.\frac{x-y}{2}\right) + \left(\frac{y+x}{2}.\frac{y-x}{2}\right)}{2} = 0.$$

37. (c) : Let the sides of the triangle be $a - d$, a, $a + d$, then hypotenuse being the greatest side *i.e.*, $a + d$.

So, $(a+d)^2 = a^2 + (a-d)^2$

$\Rightarrow \quad a^2 + d^2 + 2ad = a^2 + a^2 - 2ad + d^2 \Rightarrow a = 4d$

Therefore ratio of the side $= a - d : a : a + d$
$= (4d - d) : 4d : (4d + d) = 3 : 4 : 5.$

38. (b) : Let three numbers are $a - d$, a, $a + d$.
Given $a-d+a+a+d=15 \Rightarrow a=5$
and $(a-d)^2+a^2+(a+d)^2=83$
$\Rightarrow a^2+d^2-2ad+a^2+a^2+d^2+2ad=83$
$\Rightarrow 2(a^2+d^2)+a^2=83$
Putting $a = 5 \Rightarrow 2(25+d^2)+25=83 \Rightarrow d=2$
Thus numbers are 3, 5, 7.

39. (b) : Let four arithmetic means are A_1, A_2, A_3 and A_4. So, $3, A_1, A_2, A_3, A_4, 23$
$\Rightarrow T_6=23=a+5d \Rightarrow d=4$
Thus $A_1=3+4=7, A_2=7+4=11,$
$A_3 = 11 + 4 = 15$, $A_4 = 15 + 4 = 19$

40. (b) : Since $\frac{1}{p+q}, \frac{1}{r+p}$ and $\frac{1}{r+q}$ are in A.P.
$\therefore \frac{1}{r+p}-\frac{1}{p+q}=\frac{1}{q+r}-\frac{1}{r+p}$
$\Rightarrow \frac{p+q-r-p}{(r+p)(p+q)}=\frac{r+p-q-r}{(q+r)(r+p)}$
$\Rightarrow \frac{q-r}{p+q}=\frac{p-q}{q+r}$ or $q^2-r^2=p^2-q^2$
Hence, p^2, q^2, r^2 are in A.P.

41. (d)

42. (a, c) : Given, a, b, c are in A.P. $\therefore a + c = 2b$
Also, $a + b > c \Rightarrow 3b > 2c$
And $b + c > a \Rightarrow 2c > b \Rightarrow \frac{2}{3}<\frac{b}{c}<2$
Similarly for $\frac{b}{a}$, we get $\frac{2}{3}<\frac{b}{a}<2$

43. (a, b, c) : $S_r=r+r\left(\frac{1}{r+1}\right)+r\left(\frac{1}{r+1}\right)^2+....+\infty$
$$=\frac{r}{1-\frac{1}{r+1}}=r+1$$
$\therefore$ a, b, c are correct and d is false.

44. (a, b, c) : $\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$
$$=\frac{1}{6}n.(n+1)[2n+1+3]=\frac{n(n+1)(n+2)}{3}$$
$a = 0$, $b = 1$, $c = 2$

45. (a, b, c, d) : As a_{912} and a_{480} are divisible by 3, none of them is prime, and for a_{91}, we have
$$a_{91}=\frac{10^{91}-1}{10-1}=\frac{10^{91}-1}{10^7-1}\times\frac{10^7-1}{10-1}$$
$= (1 + 10^7 + ... + 10^{84}) (1 + 10 + ... + 10^6)$
$\Rightarrow$ a_{91} is not prime. Hence a,b,c,d are correct

46. (a, b, c, d) **47. (a, c)**

48. (b) : $G_1G_2....G_n=\left(\sqrt{1\times1024}\right)^n=2^{5n}$
$\therefore 2^{5n} = 2^{45} \Rightarrow n = 9$

49. (b) : $A_1+A_2+A_3+...+A_{m-1}+A_m=1025\times171$
$\therefore m\left(\frac{-2+1027}{2}\right)=1025\times171$
$\therefore m = 342$

50. (a) : Since $n = 9$,
$\therefore$ Common ratio $(r) = (1024)^{\frac{1}{9+1}} = 2$
$\therefore G_1 = 2, r = 2$
$G_1+G_2+...+G_n=\frac{2\cdot(2^9-1)}{2-1}=1024-2=1022$

51. (a) : Common difference of sequence
$A_1, A_2, ..., A_m$ is $\frac{1027+2}{342+1}=3$
Common difference of sequence $A_1, A_3, A_5, ..., A_{n-1}$ is 6

52. (a) : We have $A_{171} + A_{172} = -2 + 1027 = 1025$
$\therefore \frac{2A_{171}+2A_{172}}{2}=1025$
Also $G_5 = 1 \times 2^5 = 32$
$\therefore G_5^2=1024$ *i.e.* $G_5^2+1=1025$
$\therefore 2A_{171}, G_5^2+1, 2A_{172}$ are in A.P.

53. (c) **54. (b)**

55. (c) : Let numbers in set A be $a - D$, a, $a + d$ and in set B be $b - d$, b, $b + d$
$3a = 3b = 15 \Rightarrow a = b = 5$
Set $A = \{5 - D, 5, 5 + D\}$; Set $B = \{5 - d, 5, 5 + d\}$
Where $D = d + 1$
$\frac{p}{q}=\frac{5(25-D^2)}{5(25-d^2)}=\frac{7}{8}$
$25(8 - 7) = 8(d + 1)^2 - 7d$
$\Rightarrow d = -17, 1$ but $d > 0 \quad \therefore d = 1$
So numbers in set A are 3, 5, 7
Numbers in set B are 4, 5, 6
Now, $p = 3 \times 5 \times 7 = 105$
$q = 4 \times 5 \times 6 = 120$
Value of $D + d = 3$

56. (A) → (s), (B) → (r), (C) → (s), (D) → (r)
(A) $b + c = a + d = 2 \cdot 10$
$\Rightarrow a + b + c + d = 40$
(B) $(ar^2)^2 = a^2 + a^2r^2$ [where $a = 2$]
$\therefore r^4 = 1 + r^2, \quad r^4 - r^2 - 1 = 0$ [Let $r^2 = t$]
$t^2 - t - 1 = 0$

$\therefore \quad t = \frac{1 \pm \sqrt{5}}{2} \Rightarrow t = \frac{1+\sqrt{5}}{2}$

$\left(\frac{1-\sqrt{5}}{2} \text{ rejected}\right)$ gives negative values

$\therefore \quad r^2 = \frac{1+\sqrt{5}}{2}$

$\therefore$ Hypotenuse is $2\left(\frac{1+\sqrt{5}}{2}\right) = 1 + 5 = a + \sqrt{b}$

On comparing we get

$\therefore \quad a^2 + b^2 = 1 + 25 = 26$

(C) Let a, ar, ar^2 are in G.P. where $|r| < 1$

$a + ar + ar^2 = 70$

$\therefore \quad 10ar = 4a + 4ar^2$ or $10r = 4 + 4r^2$

$2r^2 - 5r + 2 = 0$ or $(2r - 1)(r - 2) = 0$

$\therefore \quad r = 1/2 \quad (\because \ |r| < 1)$

For $r = 1/2$, $a(1 + r + r^2) = 70$

$\Rightarrow \quad a\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 70 \Rightarrow a = 40$

$\therefore$ Series becomes 40, 20, 10 where first term is 40.

(D) Using cosine rule

$a^2 = 9 + 4 + 2 \cdot 2 \cdot 3 \cdot = 13 + 6 = 19$

$\therefore \quad a^2 = 19$

Similarly, $b^2 = 9 + 4 - 2 \cdot 2 \cdot 3 = 7$

$\therefore \quad b^2 = 7$

$\therefore \quad P = 2(\sqrt{19} + \sqrt{7}) = 2(\sqrt{p} + \sqrt{q})$

$\therefore \quad p + q = 19 + 7 = 26$

57. (A) → (r), (B) → (p), (C) → (s), (D) → (q)

(A) $F(n+1) = \frac{2F(n)+1}{2} = F(n) + \frac{1}{2}$

$\therefore$ $F(1)$, $F(2)$, $F(3)$, ... is an A.P. with common difference 1/2

Hence $F(101) = 2 + (100)\frac{1}{2} = 52$

(B) $a_1 + 2d + a_1 + 4d + a_1 + 10d + a_1 + 16d + a_1 + 18d$

$= 5a_1 + 50d \Rightarrow a_1 + 10d = 2$

Now, $\sum_{i=1}^{21} a_i = \frac{21}{2}[2a_1 + 20d] = 21(a_1 + 10d) = 42$

(C) $S = 1 + 5 + 13 + 29 + \ldots +$

$S = \quad 1 + 5 + 13 + \ldots +$

On subtracting, we get $t_{10} = 1 + 4 + 8 + 16 + \ldots$ up to 10 terms

$= 1 + (4 + 8 + 16 + \ldots$ up to 9 terms$) = 2045$

(D) Sum of all two digit numbers

$= \frac{90}{2}(10 + 99) = (45)(109)$

Sum of all two digit numbers divisible by 2

$= \frac{45}{2}(10 + 98) = (45)(54)$

Sum of all two digit numbers divisible by 3

$= \frac{30}{2}(12 + 99) = 15(111).$

Sum of all two digit numbers is divisible by 6

$= \frac{15}{2}(12 + 96) = 15(54)$

$\therefore$ Required sum

$= 45(109) + 15(54) - (45)(54) - 15(111) = 1620$

58. (A) → (q), (B) → (r), (C) → (p), (D) → (s)

(A) Let the number be a, b : $a + b = 12$ and

$ab + \frac{6ab}{a+b} = 48$ (Given)

$ab + \frac{ab}{2} = 48 \quad \therefore \quad ab = 32$

(B) $S = \frac{5}{1^2 \cdot 4^2} + \frac{11}{4^2 \cdot 7^2} + \frac{17}{7^2 \cdot 10^2} + \ldots$

$\Rightarrow \quad 3S = \frac{3 \cdot 5}{1^2 \cdot 4^2} + \frac{3 \cdot 11}{4^2 \cdot 7^2} + \frac{3 \cdot 17}{7^2 \cdot 10^2} + \ldots$

$\Rightarrow \quad 3S = \frac{4^2 - 1^2}{1^2 \cdot 4^2} + \frac{7^2 - 4^2}{4^2 \cdot 7^2} + \frac{10^2 - 7^2}{7^2 \cdot 10^2} + \ldots$

$\Rightarrow \quad 3S = 1 - \frac{1}{4^2} + \frac{1}{4^2} - \frac{1}{7^2} + \frac{1}{7^2} - \frac{1}{10^2} + \ldots$

$\Rightarrow \quad 3S = 1 \Rightarrow S = \frac{1}{3}$

(C) H.M of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ is $\frac{4}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}} = \frac{240}{77}$

(D) $\text{G.M.} = \sqrt{(4) \times (9)} = 6$

59. (2) : Let the series be a, ax, ax^2, ax^3, ...

Also, $\frac{T_4}{T_2} = \frac{ax^3}{ax} = \frac{1}{16} \Rightarrow x^2 = \frac{1}{16} \Rightarrow x = \pm\frac{1}{4}$

But since it is a decreasing G.P. $\therefore \ x = \frac{1}{4}$

Also, $\frac{T_3}{T_2^2} = \frac{ax^2}{(ax)^2} = \frac{1}{9} \Rightarrow \frac{1}{a} = \frac{1}{9} \Rightarrow a = 9$

$S_\infty = \frac{a}{1-r} = \frac{9}{1 - \frac{1}{4}} = \frac{9 \times 4}{3} = 12$

60. (2) : $f(x)$ is increasing.

So its greatest value is $f(3) = 27$.

Let the G.P. be a, ar, ar^2, ... with, $-1 < r < 1$

$\frac{a}{1-r} = 27$ and $a - ar = 3$

On solving, we get $r = 2/3$

CLASS XI | **Series 6**

ACE YOUR WAY CBSE

Sequences and Series

IMPORTANT FORMULAE

Arithemetic Progression (A.P.)

- *Common difference* $(d) = a_{n+1} - a_n \; \forall \; n \in N$
- *General term,* $a_n = n^{th}$ *term* $= a + (n-1)\,d$
- n^{th} *term from the end consisting of m terms is* $(m - n + 1)^{th}$ *term from the beginning*
  $\therefore$ n^{th} *term from the end* $= a_{m-n+1}$

Sum of n terms of an A.P.

- $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$
  where l is last term
- $a_n = S_n - S_{n-1}$
- $A = \frac{a+b}{2}$ *where A is A.M. of a and b*
- *If* $a_1, a_2, a_3, \ldots, a_n$ *are n numbers then*
  Arithmetic mean $= A = \frac{a_1 + a_2 + \ldots + a_n}{n}$

Geometric Progression (G.P.)

- *Common ratio* $= r = \frac{a_{n+1}}{a_n} \forall n \in N$
- *General term* $a_n = n^{th}$ *term* $= ar^{n-1}$
- n^{th} *term from the end consisting of m terms is* $(m - n + 1)^{th}$ *term from the beginning*
  $\therefore$ n^{th} *term from the end* $= a_{m-n+1} = ar^{m-n}$
  $= l\left(\frac{1}{r}\right)^{n-1}$ *where l is last term of G.P.*

Sum of n terms of the G.P.

- *Sum of finite G.P.* $= S_n = \frac{a(r^n - 1)}{r-1} = \frac{lr - a}{r-1} \; (r > 1)$
  $= \frac{a(1-r^n)}{1-r} = \frac{a - lr}{1-r} \; (r < 1)$
- *Sum of an infinite* $= S = \frac{a}{1-r} \; (|r| < 1)$

Geometric mean (G.M.)

- *If a, x, b are in G.P., then* $x = \sqrt{ab}$ *is the G.M. of a and b.*
- *If* $a, x_1, x_2, \ldots, x_n, b$ *are in G.P., then* $x_1, x_2, \ldots, x_n$ *are the n geometric means between a and b.*

$$r = \left(\frac{b}{a}\right)^{1/(n+1)} \;\&\; x_1 = ar = a\left(\frac{b}{a}\right)^{1/(n+1)}, x_2 = ar^2$$

$$= a\left(\frac{b}{a}\right)^{2/(n+1)}, \ldots\ldots, x_n = \frac{b}{r} = b\left(\frac{a}{b}\right)^{1/(n+1)}$$

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Find the n^{th} term of the series $2\frac{1}{2}+1\frac{7}{13}+1\frac{1}{9}+\frac{20}{23}+...$
2. Find the sum of first six terms of the series:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+............$$

3. If x, y, z are distinct positive numbers, then show that $(x+y)(y+z)(z+x) > 8xyz$
4. If the 9^{th} term of an A.P. vanishes, then find the ratio of its 29^{th} and 19^{th} terms.
5. Show that the product of n terms, where n is odd, of a G.P. will be equal to n^{th} power of the middle term.

SHORT ANSWER TYPE

6. If a, b, c, d are four distinct positive numbers in A.P. then show that $bc > ad$.
7. If $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ are in A.P. whose common difference is d, then show that $\sin d[\sec\alpha_1 \sec\alpha_2 + \sec\alpha_2 \sec\alpha_3 + ... + \sec\alpha_{n-1}\sec\alpha_n] = \tan\alpha_n - \tan\alpha_1$.
8. If x, y, z are positive then find the minimum value of $x^{\log y-\log z} + y^{\log z-\log x} + z^{\log x-\log y}$.
9. Find the sum : $\sum_{r=1}^{n}\frac{1}{(ar+b)(ar+a+b)}$.
10. The vibrations of system are damped so that the amplitudes of the successive deflections are 12, 8, 16/3, ... Find the amplitude of the 6^{th} deflection and also the total deflection before the system comes to rest.

LONG ANSWER TYPE - I

11. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and

$$(a_1+a_n)\left[\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\frac{1}{a_3a_{n-2}}+...+\frac{1}{a_na_1}\right]=k\sum_{r=1}^{n}\frac{1}{a_r}$$

and a be the one A.M. and x, y be the two G.M.s between b and c, then show that $x^3 + y^3 = kabc$.

12. If p^{th} and q^{th} terms of G.P. are q and p respectively, show that the $(p+q)^{th}$ term is $(q^p/p^q)^{1/(p-q)}$.
13. Find the sum upto n terms of the series:

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+......$$

14. One side of an equilateral triangle is 24 cm. The mid-points of its sides are joined to form still another triangle. This process continues, indefinitely. Find the sum of the perimeters of all the triangles.
15. If a, b, c are in A.P. and x, y, z are in G.P., then show that $x^{b-c}\cdot y^{c-a}\cdot z^{a-b} = 1$.

LONG ANSWER TYPE - II

16. Find the sum to n terms of the series:

3 + 15 + 35 + 63 + ...

17. If a, b, c are in G.P. and $\log_c a, \log_b c$ and $\log_a b$ are in A.P. show that the common difference of A.P. is 3/2.
18. If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then prove that the sum of $(m+n)$ terms is $-(m+n)$. Also, find the sum of first $(m-n)$ terms $(m > n)$.
19. Suppose x and y are two real numbers such that the r^{th} mean between x and $2y$ is equal to the r^{th} mean between $2x$ and y when n arithmetic means are inserted bwteen them in both the cases. Show that $\frac{n+1}{r}-\frac{y}{x}=1$.
20. The ratio of the sum of n terms of two A.P.'s is $(7n+1):(4n+27)$. Find the ratio of their n^{th} terms.

SOLUTIONS

1. Reciprocals of the given terms of the series are:

$$\frac{2}{5},\frac{13}{20},\frac{9}{10},\frac{23}{20},... \text{ or } \frac{8}{20},\frac{13}{20},\frac{18}{20},\frac{23}{20},...$$

$$\therefore\ n^{th}\text{ term}(a_n)=\frac{8+(n-1)5}{20}=\frac{5n+3}{20}$$

Hence, n^{th}term of the required series $=\frac{20}{5n+3}$

2. Let T_r be the rth term of the given series. Then,

$$T_r=\frac{1}{r(r+1)}, r=1,\ 2,...,n$$

$$\Rightarrow\ T_r=\frac{1}{r}-\frac{1}{r+1}, r=1,\ 2,...,n$$

$$\therefore\ \text{Required term}=\sum_{r=1}^{6}T_6=\sum_{r=1}^{6}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{6}-\frac{1}{7}\right)$$

$$=1-\frac{1}{7}=\frac{6}{7}$$

3. Using A.M. > G.M., we obtain

$$\frac{x+y}{2}>\sqrt{xy},\ \frac{y+z}{2}>\sqrt{yz}\text{ and }\frac{z+x}{2}>\sqrt{zx}$$

$$\Rightarrow\ x+y>2\sqrt{xy},\ y+z>2\sqrt{yz}\ \text{ and }\ z+x>2\sqrt{zx}$$

$\Rightarrow\ (x+y)(y+z)(z+x) > 2\sqrt{xy}\times 2\sqrt{yz}\times 2\sqrt{zx}$

$\Rightarrow\ (x+y)(y+z)(z+x) > 8xyz.$

4. $T_9 = 0 \Rightarrow a + 8d = 0 \Rightarrow a = -8d$

$\therefore\ \dfrac{T_{29}}{T_{19}} = \dfrac{a+28d}{a+18d} = \dfrac{-8d+28d}{-8d+18d} = \dfrac{20d}{10d} = \dfrac{2}{1}$

5. Let the G.P. be $a, ar, ar^2, ar^3 \ldots, ar^{n-1}$ Here, the number of terms is odd.

$\therefore$ The middle term $= ar^{((n+1)/2)-1} = ar^{(n-1)/2}$.

Then, $a \cdot ar \cdot ar^2 \cdot ar^3 \ldots\ldots ar^{n-1} = a^n r^{1+2+3+\ldots+(n-1)}$

$= a^n r^{(n-1)n/2} = [ar^{(n-1)/2}]^n =$ (the middle term)^n.

6. Given a, b, c, d are in A.P.

$\Rightarrow$ b is the A.M. of a and c

And, G.M. of a and c is $\sqrt{ac}$.

$\because$ A.M. of a and c > G.M. of a and c

$\therefore\ b > \sqrt{ac} \Rightarrow b^2 > ac$ ...(i)

Similarly, c is the A.M. of b and d

$\therefore\ c > \sqrt{bd} \Rightarrow c^2 > bd$ ..(ii)

From (i) and (ii), we get

$b^2c^2 > (ac)(bd) \Rightarrow bc > ad.$

7. We have, $\sin d\,(\sec\alpha_1 \sec\alpha_2) = \dfrac{\sin(\alpha_2-\alpha_1)}{\cos\alpha_1\cdot\cos\alpha_2}$

$= \dfrac{\sin\alpha_2\cos\alpha_1}{\cos\alpha_1\cdot\cos\alpha_2} - \dfrac{\cos\alpha_2\sin\alpha_1}{\cos\alpha_1\cdot\cos\alpha_2} = \tan\alpha_2 - \tan\alpha_1$

Similarly, $\sin d \sec\alpha_2 \sec\alpha_3 = \tan\alpha_3 - \tan\alpha_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\sin d \sec\alpha_{n-1}\sec\alpha_n = \tan\alpha_n - \tan\alpha_{n-1}$

On adding, we get

$\sin d\,[\sec\alpha_1\sec\alpha_2 + \sec\alpha_2\sec\alpha_3 + \ldots + \sec\alpha_{n-1}\sec\alpha_n]$

$= \tan\alpha_n - \tan\alpha_1$

8. Since A.M. $\ge$ G.M.

$$\Rightarrow \frac{x^{\log y-\log z} + y^{\log z-\log x} + z^{\log x-\log y}}{3}$$

$$\ge \sqrt[3]{x^{\log y-\log z}\cdot y^{\log z-\log x}\cdot z^{\log x-\log y}} = 1$$

because $\log(x^{\log y-\log z}\cdot y^{\log z-\log x}\cdot z^{\log x-\log y})$

$= (\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)\log z = 0$

$\therefore\ x^{\log y-\log z}\cdot y^{\log z-\log x}\cdot z^{\log x-\log y} = e^0 = 1$

Hence, minimum value of given expression is 1.

9. We have, $\displaystyle\sum_{r=1}^{n}\frac{1}{(ar+b)(ar+a+b)}$

$$= \sum_{r=1}^{n}\frac{1}{a}\left(\frac{1}{(ar+b)} - \frac{1}{(ar+a+b)}\right)$$

$$= \frac{1}{a}\left[\left(\frac{1}{a+b} - \frac{1}{2a+b}\right) + \left(\frac{1}{2a+b} - \frac{1}{3a+b}\right) + \ldots\right.$$

$$\left.\ldots + \left(\frac{1}{na+b} - \frac{1}{(n+1)\,a+b}\right)\right]$$

$$= \frac{1}{a}\left\{\frac{1}{a+b} - \frac{1}{(n+1)a+b}\right\} = \frac{n}{(a+b)\{(n+1)a+b\}}.$$

10. The amplitudes of the successive deflections are 12, 8, 16/3, ... which form the G.P.

Here, $a = 12,\ r = \dfrac{8}{12} = \dfrac{2}{3}$

$\therefore$ Amplitude of the 6th deflection $= ar^{6-1} = ar^5$

$= 12\times\left(\dfrac{2}{3}\right)^5 = 12\times\dfrac{32}{243} = \dfrac{128}{81}$

Total deflection before the system comes to rest = Sum to infinite terms of the G.P. $= \dfrac{a}{1-r} = \dfrac{12}{1-\frac{2}{3}} = 36$

11. Given, $(a_1+a_n)\left[\dfrac{1}{a_1a_n} + \dfrac{1}{a_2a_{n-1}} + \dfrac{1}{a_3a_{n-2}} + \ldots + \dfrac{1}{a_na_1}\right]$

$$= \frac{a_1+a_n}{a_1a_n} + \frac{a_2+a_{n-1}}{a_2a_{n-1}} + \frac{a_3+a_{n-2}}{a_3a_{n-2}} + \ldots + \frac{a_n+a_1}{a_na_1}$$

$$= \left(\frac{1}{a_n}+\frac{1}{a_1}\right) + \left(\frac{1}{a_{n-1}}+\frac{1}{a_2}\right) + \left(\frac{1}{a_{n-2}}+\frac{1}{a_3}\right) + \ldots + \left(\frac{1}{a_1}+\frac{1}{a_n}\right)$$

$$= 2\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}\right) = 2\sum_{r=1}^{n}\frac{1}{a_r}$$

$\therefore\ k = 2$

Now, $x^3 + y^3 = b^3r^3(1+r^3) = b^3\cdot\dfrac{c}{b}\left(1+\dfrac{c}{b}\right)$

$= bc\,(b+c) = bc(2a) = 2abc = kabc$

12. Let a be the first term and r be the common ratio of G.P. then, $t_p = ar^{p-1} = q$ ... (i) and $t_q = ar^{q-1} = p$ ... (ii)

Dividing (i) by (ii), we get $\dfrac{ar^{p-1}}{ar^{q-1}} = \dfrac{q}{p}$

$\Rightarrow r^{p-q} = \dfrac{q}{p} \Rightarrow r = \left(\dfrac{q}{p}\right)^{1/(p-q)}$

Now, $t_{p+q} = ar^{p+q-1} = ar^{p-1}\,r^q$

$$= q\left(\frac{q}{p}\right)^{q/(p-q)} = q\cdot\frac{q^{q/(p-q)}}{p^{q/(p-q)}} = \frac{q^{1+q/(p-q)}}{p^{q/(p-q)}}$$

$$= \frac{q^{p/(p-q)}}{p^{q/(p-q)}} = \left(\frac{q^p}{p^q}\right)^{1/(p-q)}$$

13. Let T_r be the r^{th} term of the given series. Then,

$$T_r = \frac{r}{1+r^2+r^4}, r = 1, 2, 3, \ldots, n$$

$$\Rightarrow T_r = \frac{r}{(r^2+r+1)(r^2-r+1)}$$

$$= \frac{1}{2}\left\{\frac{2r}{(r^2+r+1)(r^2-r+1)}\right\}$$

$$\Rightarrow T_r = \frac{1}{2}\left\{\frac{1}{(r^2-r+1)} - \frac{1}{(r^2+r+1)}\right\}, r = 1, 2, \ldots, n$$

$$\therefore S_n = \sum_{r=1}^{n} T_r = \frac{1}{2}\left\{\sum_{r=1}^{n}\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)\right\}$$

$$= \frac{1}{2}\left\{\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right) + \ldots + \left(\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right)\right\}$$

$$= \frac{1}{2}\left\{1 - \frac{1}{n^2+n+1}\right\} = \frac{n^2+n}{2(n^2+n+1)}.$$

14. Perimeter of the 1st equilateral $\Delta ABC = 3 \times$ side $= 3 \times 24 = 72$ cm.

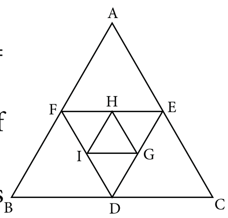
Let D, E, F are the mid-points of the sides BC, CA, AB respectively.

$$\therefore DE = \frac{1}{2}AB = \frac{1}{2} \times 24 = 12\text{cm}$$

Similarly, $EF = DF = 12$ cm

$\therefore$ Perimeter of $\Delta DEF = 3 \times 12 = 36$ cm

Similarly we get, perimeter of $\Delta GHI = 3 \times 6 = 18$ cm.



Sum of perimeters of these triangles which are infinite in number $= 72 + 36 + 18 + \ldots\ldots \infty$. (which forms the G.P.) $= \dfrac{72}{1-\frac{1}{2}}$

$= 144$ cm

15. If a, b, c are in A.P. $\Rightarrow 2b = a + c$ ...(i)

x, y, z are in G.P. $\Rightarrow y^2 = xz$ ...(ii)

$$\therefore x^{b-c}\, y^{c-a}\, z^{a-b} = x^{b-c}(\sqrt{xz})^{c-a} z^{a-b} \quad \text{[Using(ii)]}$$

$$= x^{b-c}x^{\frac{c-a}{2}} z^{\frac{c-a}{2}} z^{a-b} = x^{b-c+\frac{c-a}{2}} z^{a-b+\frac{c-a}{2}}$$

$$= x^{\frac{2b-(a+c)}{2}} z^{\frac{(c+a)-2b}{2}} = x^0 z^0 = 1$$

16. The difference between the successive terms are $15 - 3 = 12$, $35 - 15 = 20$, $63 - 35 = 28$, ... Clearly, these differences are in A.P.

Let T_n be the n^{th} term and S_n denote the sum to n terms of the given series. Then,

$S_n = 3 + 15 + 35 + 63 + \ldots + T_{n-1} + T_n$ ...(i)

$S_n = 3 + 15 + 35 + \ldots\ldots + T_{n-1} + T_n$ ...(ii)

Subtracting (ii) from (i), we get

$0 = 3 + \{12 + 20 + 28 + \ldots + (T_n - T_{n-1})\} - T_n$

$$\Rightarrow T_n = 3 + \frac{(n-1)}{2}\{2 \times 12 + (n-1-1) \times 8\}$$

$$= 3 + (n-1)(12 + 4n - 8)$$

$$\Rightarrow T_n = 3 + (n-1)(4n+4) = 4n^2 - 1$$

$$\therefore S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}(4k^2-1)$$

$$\Rightarrow S_n = 4\sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} 1 = 4\left\{\frac{n(n+1)(2n+1)}{6}\right\} - n$$

$$= \frac{n}{3}(4n^2+6n-1)$$

17. Let $x = \log_c a$, $y = \log_b c$, $z = \log_a b$ $\therefore xyz = 1$ ...(i)

Given x, y, z are in A.P. $\therefore$ $2y = x + z$ ...(ii)

Given a, b, c are in G.P.

$\therefore$ $b^2 = ac \Rightarrow 2\log_b b = \log_b a + \log_b c \Rightarrow 2 = \frac{1}{z} + y$ ...(iii)

From (ii) and (iii), $2\left(2 - \frac{1}{z}\right) = x + z$

$$\Rightarrow x = 4 - \frac{2}{z} - z = \frac{4z - 2 - z^2}{z}$$

From (i), $\dfrac{4z-2-z^2}{z} \cdot \left(2 - \dfrac{1}{z}\right) \cdot z = 1$

$\Rightarrow (4z - 2 - z^2)(2z - 1) = z$

$\Rightarrow 8z^2 - 4z - 2z^3 - 4z + 2 + z^2 = z$

$\Rightarrow -2z^3 + 9z^2 - 9z + 2 = 0$ $[\because z - 1 \neq 0]$

$\Rightarrow 2z^2 - 7z + 2 = 0$

$\therefore$ Common difference of A.P. $= z - y = z - 2 + \dfrac{1}{z}$

$$= \frac{z^2-2z+1}{z} = \frac{2z^2-4z+2}{2z} = \frac{3z}{2z} = \frac{3}{2}.$$

MPP-6 CLASS XI ANSWER KEY

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(c)	**5.**	(a)
6.	(b)	**7.**	(a,b)	**8.**	(a,b,c)	**9.**	(b,c)	**10.**	(b,d)
11.	(c)	**12.**	(b,d)	**13.**	(d)	**14.**	(b)	**15.**	(a)
16.	(a)	**17.**	(9)	**18.**	(4)	**19.**	(5)	**20.**	(4)

18. Let a be the first term and d be the common difference of the given A.P. Then,

$S_m = n \Rightarrow \frac{m}{2}\{2a+(m-1)\ d\} = n$

$\Rightarrow 2am + m(m - 1)\ d = 2n$ ...(i)

Also, $S_n = m \Rightarrow \frac{n}{2}\{2a+(n-1)\ d\} = m$

$\Rightarrow 2an + n(n - 1)\ d = 2m$ ...(ii)

Subtracting (ii) from (i), we get

$2a(m - n) + \{m(m - 1) - n(n - 1)\}\ d = 2n - 2m$

$\Rightarrow 2a(m - n) + \{(m^2 - n^2) - (m - n)\}\ d = -2(m - n)$

$\Rightarrow 2a + (m + n - 1)\ d = -2$ ... (iii)

Now, $S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)\ d\}$

$\Rightarrow S_{m+n} = \frac{(m+n)}{2}(-2)\ \therefore S_{m+n} = -(m+n)$

From (iii), we obtain

$2a = -2 - (m + n - 1)d$ ... (iv)

Substituting this value of $2a$ in (i), we obtain

$-2m - m(m + n - 1)\ d + m(m - 1)\ d = 2n$

$\Rightarrow d = -2\left(\frac{m+n}{mn}\right)$

Putting $d = -2\left(\frac{m+n}{mn}\right)$ in (iv), we obtain

$2a = -2 + \frac{2}{mn}(m+n-1)(m+n)$

Now, $S_{m-n} = \frac{m-n}{2}\{2a+(m-n-1)\ d\}$

$$\Rightarrow S_{m-n} = \frac{m-n}{2}\left\{-2+\frac{2}{mn}(m+n-1)(m+n) - \frac{2}{mn}(m-n-1)(m+n)\right\}$$

$$\Rightarrow S_{m-n} = \frac{m-n}{2}\left\{-2+\frac{4n}{mn}(m+n)\right\} = \frac{1}{m}(m-n)(m+2n)$$

19. Let $A_1, A_2, \ldots, A_n$ be n arithmetic means between x and $2y$. Then, $x, A_1, A_2, \ldots, A_n, 2y$ are in A.P. with common difference d_1 given by $d_1 = \frac{2y-x}{n+1}$.

$\therefore\ r^{\text{th}}$ mean $= A_r = x + rd_1 = x + r\left(\frac{2y-x}{n+1}\right)$

Let $A'_1, A'_2, \ldots, A'_n$ be n arithmetic means between $2x$ and y. Then, $2x, A'_1, A'_2, \ldots, A'_n, y$ are in A.P. with common difference d_2 given by $d_2 = \frac{y-2x}{n+1}$.

$\therefore\ r^{\text{th}}$ mean $= A_r = 2x + rd_2 = 2x + r\left(\frac{y-2x}{n+1}\right)$

It is given that: $A_r = A'_r$

$\Rightarrow x + r\left(\frac{2y-x}{n+1}\right) = 2x + r\left(\frac{y-2x}{n+1}\right)$

$\Rightarrow (n + 1)\ x + r\ (2y - x) = (n + 1)\ 2x + r(y - 2x)$

$\Rightarrow (n + 1)\ x - ry = rx$

$\Rightarrow \frac{n+1}{r} - \frac{y}{x} = 1$

20. Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P.'s respectively. Then, the sums S_n and S'_n of their n terms are given by

$S_n = \frac{n}{2}\{2a_1+(n-1)\ d_1\}$ and $S'_n = \frac{n}{2}\{2a_2+(n-1)\ d_2\}$

$$\therefore \frac{S_n}{S'_n} = \frac{\frac{n}{2}\{2a_1+(n-1)\ d_1\}}{\frac{n}{2}\{2a_2+(n-1)\ d_2\}} = \frac{\{2a_1+(n-1)\ d_1\}}{\{2a_2+(n-1)\ d_2\}}$$

It is given that $\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$$\Rightarrow \frac{2a_1+(n-1)\ d_2}{2a_2+(n-1)\ d_2} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}\ d_2}{a_2+\frac{(n-1)}{2}\ d_2} = \frac{7n+1}{4n+27} \qquad \text{...(i)}$$

Since, the ratio to m^{th} terms of two A.P.'s is, $\frac{a_1+(m-1)\ d_2}{a_2+(m-1)\ d_2}$

So, replacing $\frac{n-1}{2}$ by $(m - 1)$ on the L.H.S of (i) we get

i.e., $\frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$.

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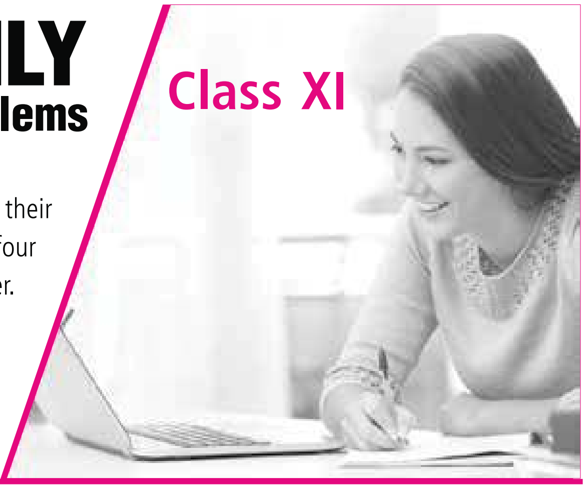
MPP-6 MONTHLY Practice Problems

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Sequences and Series

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If a, b, c are positive reals, then least value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is

(a) 1 (b) 6
(c) 9 (d) none of these

2. Two A.M's A_1 and A_2 ; two G.M's G_1 and G_2 and two H.M's H_1 and H_2 are inserted between any two numbers, then $H_1^{-1} + H_2^{-1}$ equals

(a) $A_1^{-1} + A_2^{-1}$ (b) $G_1^{-1} + G_2^{-1}$
(c) $\frac{G_1G_2}{(A_1+A_2)}$ (d) $\frac{(A_1+A_2)}{G_1G_2}$

3. If a, b, c are digits, then the rational number represented by $0.cababab$ .... is

(a) $\frac{cab}{990}$ (b) $\frac{(99c+ba)}{990}$
(c) $\frac{(99c+10a+b)}{99}$ (d) $\frac{(99c+10a+b)}{990}$

4. If H is the harmonic mean between a and b, then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

5. If the arithmetic progression whose common difference is non zero, the sum of first $3n$ terms is equal to the sum of the next n terms. Then the ratio of the sum of the first $2n$ terms to the next $2n$ terms is

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) none of these.

6. If $3 + 5r + 7r^2 + ...$ to ∞ to $\frac{44}{9}$, then r is equal to

(a) $\frac{17}{11}$ (b) $\frac{1}{4}$ (c) $\frac{11}{17}$ (d) 4

One or More Than One Option(s) Correct Type

7. If $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P. then x is equal to

(a) $\log_a(\log_b a)$
(b) $\log_a(\log_e a) - \log_a(\log_e b)$
(c) $-\log_a(\log_a b)$
(d) $\log_a(\log_e b) - \log_a(\log_e a)$

8. The series of natural numbers is divided into groups 1 ; 2, 3, 4 ; 5, 6, 7, 8, 9 ; .... and so on. Then the sum of the numbers in the n^{th} group is

(a) $(2n-1)(n^2-n+1)$ (b) $2n^3-3n^2+3n-1$
(c) $n^3+(n-1)^3$ (d) $n^3+(n+1)^3$

9. Let a, b, c, d, e be five numbers such that a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then $b =$

(a) 2 (b) -2 (c) 4 (d) -4

10. There are two numbers a and b whose product is 192 and the quotient of A.M. by H.M. of their greatest common divisor and least common multiple is $\frac{169}{48}$. The smaller of a and b is

(a) 2 (b) 4 (c) 6 (d) 12

11. Given that $0 < x < \frac{\pi}{4}$ and $\frac{\pi}{4} < y < \frac{\pi}{2}$

and $\sum_{k=0}^{\infty}(-1)^k\tan^{2k}x = a, \sum_{k=0}^{\infty}(-1)^k\cot^{2k}y = b,$

then $\sum_{k=0}^{\infty}(\tan)^{2k}x\cot^{2k}y$ is

(a) $\frac{1}{a}+\frac{1}{b}-\frac{1}{ab}$ (b) $a+b-ab$

(c) $\dfrac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{ab}}$ (d) $\dfrac{ab}{a+b-1}$

12. The first three terms of a progression are 3, −1, −1. The next term is

(a) 2 (b) 3 (c) $\frac{-5}{27}$ (d) $-\frac{5}{9}$

13. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

Comprehension Type

Let $\{a_n\}$ be a sequence such that $\sum_{r=1}^{n} a_r = n^2$.

14. $\frac{1^3}{a_1}+\frac{1^3+2^3}{a_1+a_2}+\frac{1^3+2^3+3^3}{a_1+a_2+a_3}+\ldots$ upto 16 terms is

(a) 346 (b) 446 (c) 546 (d) 464

15. $\frac{a_1}{\lfloor\underline{1}}+\frac{a_1+a_2}{\lfloor\underline{2}}+\frac{a_1+a_2+a_3}{\lfloor\underline{3}}+\ldots$ upto ∞ is

(a) $2e$ (b) $2e-1$ (c) e^2 (d) $e-1$

Matrix Match Type

16. Match the following :

	Column I		Column II
P.	Three numbers a, b, c between 2 and 18 such that $a + b + c = 25$; 2, a, b are consecutive terms of an A.P. ; b, c, 18 are consecutive terms of an G.P. If G = Max $\{a, b, c\}$ and L = Min $\{a, b, c\}$ then	1.	$\frac{1}{2}\left(e+\frac{1}{e}\right)$
Q.	Three numbers a, b, c are in G.P. such that $a + b + c = 70$; $4a$, $5b$, $4c$ are in AP. If G = Max$\{a, b, c\}$ and L = Min $\{a, b, c\}$ then	2.	$\left[\frac{G}{L}\right]=4$ where $[\cdot]$ denotes the greatest integer function
R.	$1+\frac{1}{\lfloor\underline{3}}+\frac{1}{\lfloor\underline{5}}+\frac{1}{\lfloor\underline{7}}+\ldots$ to ∞ is equal to	3.	$G - L = 7$
		4.	$\frac{1}{2}\left(e-\frac{1}{e}\right)$

	P	Q	R
(a)	3	2	4
(b)	3	4	1
(c)	1	4	3
(d)	2	1	3

Integer Answer Type

17. If $x_1, x_2, x_3, \ldots, x_{2008}$ are in H.P. and $\sum_{i=1}^{2007} x_i x_{i+1} = \lambda x_1 x_{2008}$, then sum of the digits of λ is

18. If a, b, c are in H.P. and if $\left(\frac{a+b}{2a-b}\right)+\left(\frac{c+b}{2c-b}\right) > \sqrt{\lambda\sqrt{\lambda\sqrt{\lambda\ldots\infty}}}$, then the value of λ must be

19. The number of zeroes in the end of the product of $5^6 \times 6^7 \times 7^8 \times 8^9 \times \ldots \times 50^{51}$ must be 57λ where λ is

20. Value of $S = 1+\frac{2}{2}+\frac{3}{4}+\frac{4}{8}+\frac{5}{16}+\ldots$ to ∞ is

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CONCEPT BOOSTERS

Limits, Continuity and Differentiability

Class XII

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

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LIMIT OF A FUNCTION

Let $y = f(x)$ be a function of x. If at $x = a$ $f(x)$ takes indeterminate form, then we consider the values of the function which are very near to 'a'. If these values tend to a definite unique number as x tends to 'a', then the unique number so obtained is called the limit of $f(x)$ at $x = a$ and we write it as $\lim\limits_{x\to a} f(x)$.

- **Left hand and right hand limit :** Consider the values of the functions at the points which are very near to a on the left of a. If these values tend to a definite unique number as x tends to a, then the unique number so obtained is called left-hand limit of $f(x)$ at $x = a$ and symbolically we write it as
$\lim\limits_{x\to a^-} f(x)=\lim\limits_{h\to 0} f(a-h)$.
Similarly we can define right-hand limit of $f(x)$ at $x = a$ which is expressed as
$\lim\limits_{x\to a^+} f(x)=\lim\limits_{h\to 0} f(a+h)$
- **Existence of limit :** $\lim\limits_{x\to a} f(x)$ exists when, both $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$ exist and
$\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$ *i.e.* L.H.L. = R.H.L.

FUNDAMENTAL THEOREMS ON LIMITS

Let $\lim\limits_{x\to 0} f(x)=l$ and $\lim\limits_{x\to 0} g(x)=m$ (l and m are real numbers) then

- $\lim\limits_{x\to a}(f(x)\pm g(x))=l\pm m$
- $\lim\limits_{x\to a}(f(x)\cdot g(x))=l\cdot m$
- $\lim\limits_{x\to a} k\ f(x)=k\cdot l$
- $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{l}{m}, m\neq 0$
- If $\lim\limits_{x\to a} f(x)=+\infty$ or $-\infty$, then $\lim\limits_{x\to a}\dfrac{1}{f(x)}=0$
- $\lim\limits_{x\to a}\log\{f(x)\}=\log\{\lim\limits_{x\to a} f(x)\}$
- If $f(x)\le g(x)$ for all x, then $\lim\limits_{x\to a} f(x)\le \lim\limits_{x\to a} g(x)$
- $\lim\limits_{x\to a}[f(x)]^{g(x)}=\{\lim\limits_{x\to a} f(x)\}^{\lim\limits_{x\to a} g(x)}$
- If p and q are integers, then $\lim\limits_{x\to a}(f(x))^{p/q}=l^{p/q}$, provided $(l)^{p/q}$ is a real number.
- $\lim\limits_{x\to a} f(g(x))=f(\lim\limits_{x\to a} g(x))=f(m)$ provided 'f' is continuous at $g(x) = m$.

Trigonometric Limits

- $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1=\lim\limits_{x\to 0}\dfrac{\sin^{-1}x}{x}$
- $\lim\limits_{x\to 0}\dfrac{\tan x}{x}=1=\lim\limits_{x\to 0}\dfrac{\tan^{-1}x}{x}$
- $\lim\limits_{x\to 0}\cos x=1$
- $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=\lim\limits_{x\to\infty}\dfrac{\cos x}{x}=0$
- $\lim\limits_{x\to\infty}\dfrac{\sin(1/x)}{(1/x)}=1$

Logarithmic Limits

- $\lim\limits_{x\to 0}\dfrac{\log(1+x)}{x}=1$
- $\lim\limits_{x\to e}\log_e x=1$

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He trains IIT and Olympiad aspirants.

- $\lim_{x\to 0}\frac{\log(1-x)}{x}=-1$
- $\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\log_a e, a>0, \neq 1$

Exponential Limits

- $\lim_{x\to 0}\frac{e^x-1}{x}=1$
- $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$
- $\lim_{x\to 0}\frac{e^{\lambda x}-1}{x}=\lambda \ (\lambda\neq 0)$

Based on the Form 1^∞

- If $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$, then

  $\lim_{x\to a}\{1+f(x)\}^{1/g(x)}=e^{\lim_{x\to a}\frac{f(x)}{g(x)}}$ or when

  $\lim_{x\to a} f(x)=1$ and $\lim_{x\to a} g(x)=\infty$, then

  $\lim_{x\to a}\{f(x)\}^{g(x)}=\lim_{x\to a}[1+f(x)-1]^{g(x)}$

  $=e^{\lim_{x\to a}(f(x)-1)g(x)}$
- $\lim_{x\to 0}(1+x)^{1/x}=e$
- $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$
- $\lim_{x\to 0}(1+\lambda x)^{1/x}=e^\lambda$
- $e \lim_{x\to\infty}\left(1+\frac{\lambda}{x}\right)^x=e^\lambda$
- $\lim_{x\to\infty} a^x=\begin{cases}\infty, \text{if } a>1\\ 0, \text{if } a<1\end{cases}$

L-Hospital's Rule

If $f(x)$ and $g(x)$ be two functions of x such that

- $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$
- Both are continuous and differentiable at $x = a$.
- $f'(x)$ and $g'(x)$ are continuous at the point $x = a$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$ provided that $g'(a)\neq 0$.

  The above rule is also applicable if $\lim_{x\to a} f(x)=\infty$ and $\lim_{x\to a} g(x)=\infty$.

CONTINUITY OF A FUNCTION AT A POINT

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain if and only if $\lim_{x\to a} f(x)$ exists and is equal to $f(a)$ *i.e.*, if $\lim_{x\to a^-} f(x)=f(a)=\lim_{x\to a+} f(x)$

- **Cauchy's definition of continuity :** A function f is said to be continuous at a point a of its domain D if for every $\in\ >0$ there exists $\delta>0$ (dependent on $\in$) such that $|x-a|<\delta \Rightarrow |f(x)-f(a)|<\in$.

PROPERTIES OF CONTINUOUS FUNCTION

Let $f(x)$ and $g(x)$ be two continuous functions at $x = a$. Then

- A function $f(x)$ is said to be everywhere continuous if it is continuous on the entire real line R *i.e.* $(-\infty, \infty)$. *e.g.*, polynomial function, e^x, $\sin x$, $\cos x$, constant, x^n, $|x-a|$ etc.
- If $g(x)$ is continuous at $x = a$ and $f(x)$ is continuous at $x = g(a)$ then $(fog)(x)$ is continuous at $x = a$.
- If $f(x)$ is continuous in a closed interval $[a, b]$ then it is bounded on this interval.
- If $f(x)$ is a continuous function defined on $[a, b]$ such that $f(a)$ and $f(b)$ are of opposite signs, then there is atleast one value of x for which $f(x)$ vanishes. *i.e.* if $f(a)>0, f(b)<0 \Rightarrow \exists c\in(a,b)$ such that $f(c) = 0$.

DISCONTINUOUS FUNCTION

- **Discontinuous function :** A function 'f' which is not continuous at a point $x = a$ in its domain is said to be discontinuous there at. The point 'a' is called a point of discontinuity of the function.

  The discontinuity may arise if $\lim_{x\to a^+} f(x)$ as well as $\lim_{x\to a^-} f(x)$ both may exist, but either of the two or both may not be equal to $f(a)$.

DIFFERENTIABILITY OF A FUNCTION AT A POINT

- A function $f(x)$ is said to be differentiable (finitely) at $x = a$ if R.H.D. at $x = a$ $[f'(a^+)]$ = L.H.D. at $x = a$ $[f'(a^-)]$ = finite value

  i.e. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$ = finite

  and the common limit is called the derivative of $f(x)$ at $x = a$, denoted by $f'(a)$. Clearly, $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ {$x\to a$ from the left as well as from the right}.

SOME STANDARD RESULTS ON DIFFERENTIABILITY

- Every polynomial function is differentiable at each $x\in R$.
- The exponential function a^x, $a > 0$ is differentiable at each $x\in R$.

- Every constant function is differentiable at each $x \in R$.
- The logarithmic function is differentiable at each point in its domain.
- Trigonometric and inverse trigonometric functions are differentiable in their domains.
- The sum, difference, product and quotient of two differentiable functions is differentiable.
- The composition of differentiable function is a differentiable function.

IMPORTANT POINTS

- Any continuous function $f(x)$, which has at least one local maximum or local minimum, is many-one.
- If $\lim_{x\to a} f(x)$ does not exist, then we can not remove this discontinuity. So this become a non-removable discontinuity or essential discontinuity.
- If f is continuous at $x = c$ and g is discontinuous at $x = c$, then $f + g$ and $f - g$ are discontinuous $f{\cdot}g$ may be continuous .
- If f and g are discontinuous at $x = c$, then $f + g$, $f - g$ and $f{\cdot}g$ may still be continuous.
- Point functions (domain and range consists one value only) is not a continuous function.
- If a function is differentiable at a point, then it is continuous also at that point.
  i.e., Differentiability $\Rightarrow$ Continuity, but the converse need not be true.
- If $f(x)$ and $g(x)$ both are not differentiable at $x = a$ then the product function $f(x){\cdot}g(x)$ and sum function $f(x) + g(x)$ can still be differentiable at $x = a$.
- If $f(x)$ is differentiable at $x = a$ and $g(x)$ is not differentiable at $x = a$ then the sum function $f(x)+ g(x)$ is also not differentiable at $x = a$.

PROBLEMS

Single Correct Answer Type

1. If $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then $\lim_{x\to 0} f(x)=$

(a) 1 (b) 0 (c) -1 (d) 2

2. $\lim_{x\to 0}\dfrac{x^3\cot x}{1-\cos x}=$

(a) 0 (b) 1 (c) 2 (d) -2

3. $\lim_{n\to\infty}\dfrac{n(2n+1)^2}{(n+2)(n^2+3n-1)}=$

(a) 0 (b) 2 (c) 4 (d) ∞

4. $\lim_{x\to a}\dfrac{\sqrt{3x-a}-\sqrt{x+a}}{x-a}=$

(a) $\sqrt{2a}$ (b) $1/\sqrt{2a}$ (c) $2a$ (d) $1/2a$

5. If $f(x)=\begin{cases} x, \text{ when } 0\le x\le 1 \\ 2-x, \text{ when } 1< x\le 2 \end{cases}$, then

(a) 1 (b) 2 (c) 0 (d) -1

6. $\lim_{x\to 0}\dfrac{e^{1/x}-1}{e^{1/x}+1}=$

(a) 0 (b) 1
(c) -1 (d) does not exist

7. $\lim_{x\to 0}\dfrac{\log\cos x}{x}=$

(a) 0 (b) 1 (c) ∞ (d) 1/2

8. If $f(9) = 9$, $f'(9) = 4$, then $\lim_{x\to 9}\dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3}=$

(a) 2 (b) 4 (c) -2 (d) -4

9. $\lim_{x\to 0}\dfrac{e^{\sin x}-1}{x}=$

(a) 1 (b) e (c) $1/e$ (d) 3

10. $\lim_{\alpha\to\pi/4}\dfrac{\sin\alpha-\cos\alpha}{\alpha-\dfrac{\pi}{4}}=$

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) 1 (d) -1

11. $\lim_{x\to\pi/2}\tan x\log\sin x=$

(a) 0 (b) 1 (c) -1 (d) 2

12. $\lim_{x\to 0}\dfrac{\cos ax-\cos bx}{x^2}=$

(a) $\dfrac{a^2-b^2}{2}$ (b) $\dfrac{b^2-a^2}{2}$
(c) $a^2 - b^2$ (d) $b^2 - a^2$

13. $\lim_{x\to 0}\dfrac{(1+x)^5-1}{(1+x)^3-1}=$

(a) 0 (b) 1 (c) 5/3 (d) 3/5

14. $\lim_{x\to 0}\dfrac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}=$

(a) 1/2 (b) 1/4 (c) 2 (d) 4

15. $\lim\limits_{x\to 0} x\log(\sin x) =$

(a) -1 (b) $\log_e 1$ (c) 1 (d) log 2

16. $\lim\limits_{x\to 0}\left\{\dfrac{\sin x - x + \dfrac{x^3}{6}}{x^5}\right\} =$

(a) 1/120 (b) $-1/120$
(c) 1/20 (d) $-1/20$

17. $\lim\limits_{h\to 0}\dfrac{(a+h)^2\sin(a+h) - a^2\sin a}{h} =$

(a) $a\cos a + a^2\sin a$ (b) $a\sin a + a^2\cos a$
(c) $2a\sin a + a^2\cos a$ (d) $2a\cos a + a^2\sin a$

18. $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} =$

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{2}{3\sqrt{3}}$ (c) $\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{2}{3}$

19. $\lim\limits_{y\to 0}\dfrac{(x+y)\sec(x+y) - x\sec x}{y} =$

(a) $\sec x\,(x\tan x + 1)$ (b) $x\tan x + \sec x$
(c) $x\sec x + \tan x$ (d) $-2/3$

20. $\lim\limits_{x\to 0}\dfrac{\tan x - \sin x}{x^3} =$

(a) 1/2 (b) $-1/2$ (c) 2/3 (d) 4

21. $\lim\limits_{x\to 1}\dfrac{1-\sqrt{x}}{(\cos^{-1}x)^2} =$

(a) 1 (b) 1/2
(c) 1/4 (d) none of these

22. $\lim\limits_{x\to \pi/4}\dfrac{\sqrt{2}\cos x - 1}{\cot x - 1} =$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{2\sqrt{2}}$ (d) 1

23. $\lim\limits_{x\to 0}\left[\dfrac{x}{\tan^{-1}2x}\right] =$

(a) 0 (b) 1/2 (c) 1 (d) ∞

24. $\lim\limits_{x\to 0}\dfrac{\sqrt{\dfrac{1}{2}(1-\cos 2x)}}{x} =$

(a) 1 (b) -1
(c) 0 (d) does not exist

25. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x+1}\right)^{x+3}$ is

(a) 1 (b) e (c) e^2 (d) e^3

26. $\lim\limits_{x\to 0}\dfrac{\sin x + \log(1-x)}{x^2}$ is equal to

(a) 0 (b) 1/2 (c) $-1/2$ (d) 2

27. The value of $\lim\limits_{x\to\infty}\dfrac{x^2\sin\frac{1}{x} - x}{1-|x|}$ is

(a) 0 (b) 1 (c) -1 (d) 2

28. $\lim\limits_{x\to -1}\dfrac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is given by

(a) $\dfrac{1}{\sqrt{\pi}}$ (b) $\dfrac{1}{\sqrt{2\pi}}$ (c) 1 (d) 0

29. $\lim\limits_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is equal to

(a) 0 (b) 1/2 (c) log2 (d) e^2

30. If $f(x) = \begin{cases} \dfrac{x^2-4x+3}{x^2-1}, & \text{for } x\neq 1 \\ 2, & \text{for } x = 1\end{cases}$, then

(a) $\lim\limits_{x\to 1^+} f(x) = 2$ (b) $\lim\limits_{x\to 1^-} f(x) = 3$
(c) $f(x)$ is discontinuous at $x = 1$
(d) none of these

31. If $f(x) = \begin{cases} x-1, & x<0 \\ \dfrac{1}{4}, & x = 0 \\ x^2, & x>0\end{cases}$, then

(a) $\lim\limits_{x\to 0^+} f(x) = 1$
(b) $\lim\limits_{x\to 0^-} f(x) = 1$
(c) $f(x)$ is discontinuous at $x = 0$
(d) none of these

32. At which points the function $f(x) = \dfrac{x}{[x]}$, where $[\cdot]$ is greatest integer function, is discontinuous

(a) only positive integers
(b) all positive and negative integers and (0, 1)
(c) all rational numbers
(d) none of these

33. If $f(x)=\begin{cases} -x^2, \text{when } x\le 0 \\ 5x-4, \text{when} 0<x\le 1 \\ 4x^2-3x, \text{when } 1<x<2 \\ 3x+4, \text{when } x\ge 2\end{cases}$, then

(a) $f(x)$ is continuous at $x = 0$
(b) $f(x)$ is continuous at $x = 2$
(c) $f(x)$ is discontinuous at $x = 1$
(d) none of these

34. Let $f(x)=\begin{cases} \frac{x-4}{|x-4|}+a, x<4 \\ a+b, x=4. \\ \frac{x-4}{|x-4|}+b, x>4\end{cases}$

Then $f(x)$ is continuous at $x = 4$ when

(a) $a = 0, b = 0$ (b) $a = 1, b = 1$
(c) $a = -1, b = 1$ (d) $a = 1, b = -1$

35. The value of $f(0)$, so that the function $f(x)=\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}, (x\ne 0)$ is continuous, is given by

(a) 2/3 (b) 6 (c) 2 (d) 4

36. If the function $f(x)=\begin{cases} 1+\sin\frac{\pi x}{2}, \text{ for } -\infty<x\le 1 \\ ax+b, \text{ for } 1<x<3 \\ 6\tan\frac{\pi x}{12}, \text{ for } 3\le x<6\end{cases}$

is continuous in the interval $(-\infty, 6)$, then the values of a and b are respectively

(a) 0, 2 (b) 1, 1 (c) 2, 0 (d) 2, 1

37. The values of A and B such that the function

$f(x)=\begin{cases} -2\sin x \quad , \quad x\le -\frac{\pi}{2} \\ A\sin x+B \quad , \quad -\frac{\pi}{2}<x<\frac{\pi}{2} \\ \cos x \quad , \quad x\ge \frac{\pi}{2}\end{cases}$, is continuous everywhere are

(a) $A = 0, B = 1$ (b) $A = 1, B = 1$
(c) $A = -1, B = 1$ (d) $A = -1, B = 0$

38. Function $f(x)=\frac{1-\cos 4x}{8x^2}$, where $x \ne 0$ and $f(x) = k$ where $x = 0$ is a continous function at $x = 0$ then the value of k will be

(a) $k = 0$ (b) $k = 1$
(c) $k = -1$ (d) none of these

39. Let $f(x)=\begin{cases} x^p\sin\frac{1}{x}, x\ne 0 \\ 0 \quad , x=0\end{cases}$, then $f(x)$ is continuous but not differential at $x = 0$ if

(a) $0 < p \le 1$ (b) $1 \le p < \infty$
(c) $-\infty < p < 0$ (d) $p = 0$

40. If $f(x)=\begin{cases} x\frac{e^{(1/x)}-e^{(-1/x)}}{e^{(1/x)}+e^{(-1/x)}}, x\ne 0 \\ 0, x=0\end{cases}$ then which of the following is true?

(a) f is continuous and differentiable at every point
(b) f is continuous at every point but is not differentiable
(c) f is differentiable at every point
(d) f is differentiable only at the origin

41. If $f(x) = |x - 3|$, then f is

(a) discontinuous at $x = 2$
(b) not differentiable at $x = 2$
(c) differentiable at $x = 3$
(d) Continuous but not differentiable at $x = 3$

42. Consider $f(x)=\begin{cases} \frac{x^2}{|x|}, x\ne 0 \\ 0, x=0\end{cases}$

(a) $f(x)$ is discontinuous everywhere
(b) $f(x)$ is continuous everywhere
(c) $f'(x)$ exists in $(-1, 1)$
(d) $f'(x)$ exists in $(-2, 2)$

43. The function $y = e^{-|x|}$ is

(a) continuous and differentiable at $x = 0$
(b) neither continuous nor differentiable at $x = 0$
(c) continuous but not differentiable at $x = 0$
(d) not continuous but differentiable at $x = 0$

44. The left-hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, k is an integer and $[x]$ = greatest integer $\le x$, is

(a) $(-1)^k (k - 1)\pi$ (b) $(-1)^{k-1} (k - 1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

45. The value of m for which the function $f(x)=\begin{cases} mx^2, x\le 1 \\ 2x, x>1\end{cases}$ is differentiable at $x = 1$, is

(a) 0 (b) 1
(c) 2 (d) does not exist

46. $f(x) = ||x| - 1|$ is not differentiable at

(a) 0 (b) $\pm 1, 0$ (c) 1 (d) ± 1

Multiple Correct Answer Type

47. Let a function $f : R \to R$ satisfies the equation $f(x+y) = f(x) + f(y),\ \forall\ x, y \in R$ then
(a) f is continuous for all $x \in R$ if it is continuous at $x = 0$
(b) $f(x) = x \cdot f(1)\ \forall\ x \in R$, if '$f$' is continuous
(c) $f(x) = (f(1))^x\ \forall\ x \in R$, if '$f$' is continuous
(d) $f(x)$ is differentiable for all $x \in R$

48. Let $\phi\left(\frac{x+2y}{3}\right) = \frac{\phi(x)+2\phi(y)}{3}\ \forall x, y \in R$ and $\phi'(0) = 1$ and $\phi(0) = 2$ then
(a) $\phi(x)$ is continuous $\forall\ x \in R$
(b) $\phi(x)$ is differentiable $\forall\ x \in R$
(c) $\phi(x)$ is both continuous and differentiable
(d) $\phi(x)$ is discontinuous at $x = 0$

49. Consider the function 'ϕ' defined in [0, 1] as
$$\phi(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x}\right) & ;\ \text{if}\quad x \neq 0 \\ 0 & ;\ \text{if}\quad x = 0 \end{cases},\ \text{then}$$
(a) $\phi(x)$ has right derivate at $x = 0$
(b) $\phi'(x)$ is discontinuous at $x = 0$
(c) $\phi'(x)$ is continuous at $x = 0$
(d) $\phi'(x)$ is differentiable at $x = 0$

50. The in-circle of ΔABC touches side BC at D. Then difference between BD and CD (R is circum-radius of ΔABC)
(a) $\left|4R\sin\frac{A}{2}\sin\frac{B-C}{2}\right|$
(b) $\left|4R\cos\frac{A}{2}\sin\frac{B-C}{2}\right|$
(c) $|b-c|$
(d) $\left|\frac{b-c}{2}\right|$

51. If $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then
(a) $f'(0^+) = 1$
(b) $f'(0^+) = 0$
(c) $f'(0^-) = 1$
(d) $f'(0^-) = 0$

52. Consider the function $y = f(x) = \sqrt{1-\sqrt{1-x^2}}$. Then the true statements among the following is/are
(a) f is continuous in its domain
(b) f is differentiable in $(-1, 1)$
(c) $Rf'(0) = \sqrt{2}$ and $Lf'(0) = -\sqrt{2}$
(d) If $\pi < \theta < \frac{3\pi}{2}$ then $f'(\sin\theta) = \frac{\cos\frac{\theta}{2}}{\sqrt{2}\cos\theta}$

53. If $f(x) = |x-a|\ \phi(x)$, where $\phi(x)$ is a continuous function, then
(a) $f'(a^+) = \phi(a)$
(b) $f'(a^-) = -\phi(a)$
(c) $f'(a^+) = f'(a^-)$
(d) $f'(a)$ does not exist

54. $f(x) = \begin{cases} \left[|x|\left[\frac{1}{|x|}\right]\right], & |x| \neq \frac{1}{n}, n \in N, \\ 0, & |x| = \frac{1}{n} \end{cases}$ then, (where [·] denotes greatest integer function)
(a) f is differentiable everywhere
(b) f is continuous everywhere
(c) f is periodic
(d) f is not an odd function

55. If $f(x) = 2 + |\sin^{-1}x|$, it is
(a) continuous no where
(b) continuous everywhere in its domain
(c) differentiable no where in its domain
(d) not differentiable at $x = 0$

56. $f(x) = \cos\pi(|x| + [x])$, then f is (where [.] denotes greatest integer function)
(a) continuous at $x = 1/2$
(b) continuous at $x = 0$
(c) differentiable in $(-1, 0)$
(d) differentiable in $(0, 1)$

Comprehension Type

Paragraph for Q. No. 57 to 59

Let $p(x)$ be a polynomial with positive leading coefficient and $p(0) = 0$; and $p(p(x)) = x \cdot \int_0^x p(t)dt, \forall x \in R$. Then

57. Degree of the polynomial $p(x)$ is
(a) 4
(b) 3
(c) 5
(d) 2

58. $\frac{p'(x)}{|x|}$ is discontinuous at $x =$
(a) 0
(b) 1
(c) -1
(d) none of these

59. If $p(1) = 3$, $p(-1) = 5$ and $g(x)$ is inverse of $p'(x)$ then $g'(0)$ is
(a) is equal to 1/4
(b) is equal to 1/8
(c) is equal to 8
(d) does not exist

Paragraph for Q. No. 60 to 62

For $x > 0$, let $f(x) = \lim_{n\to\infty} \frac{\log(2+x) + x^{2^n}\sin x}{1+x^{2^n}}$

60. $\lim_{x\to 0^+} f(x)$ is equal to
(a) 0
(b) $\log_e 3$
(c) $\log_e 2$
(d) does not exist

61. At $x = 1, f'(x)$ is
(a) continuous
(b) discontinuous
(c) both continuous and differentiable
(d) continuous but not differentiable.

62. In $(0, \pi/2]$, the number of points at which $f(x)$ vanishes is
(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Q. No. 63 to 65

A function $f(x)$ is said to be continuous at $x = a$ if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$. *i.e.*, $\lim_{x\to a} f(x) = f(a)$

When $f(x)$ is not continuous at $x = a$ we say that $f(x)$ discontinuous at $x = a$.

63. If $f(x) = \lim_{m\to\infty} \sin^{2m} x,$, then number of point(s) where $f(x)$ is discontinuous is
(a) 0 (b) 1
(c) 2 (d) infinitely many

64. $\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$
(a) exists and it equals $\sqrt{2}$
(b) exists and it equals $-\sqrt{2}$
(c) does not exists because L.H.L. $\neq$ R.H.L.
(d) exists and it equals 1/2

65. In order that the function $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$, $f(0)$ must be defined as
(a) 0 (b) e (c) $1/e$ (d) 1

Matrix – Match Type

66. Match the following.

	Column I		Column II
(A)	$f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is not differentiable at	(p)	$x = 1$
(B)	$f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is not differentiable at	(q)	$x = -1$
(C)	$f(x) = \cos^{-1}(4x^3 - 3x)$ is not differentiable at	(r)	$x = 1/2$
(D)	$f(x) = \sin^{-1}(3x - 4x^3)$ is not differentiable at	(s)	$x = -1/2$

67. Match the following.

	Column I		Column II
(A)	$f(x) = \lvert x\rvert$	(p)	Continuous at $x = 0$
(B)	$f(x) = x^n \lvert x\rvert, n \in N$	(q)	Discontinuous at $x = 0$
(C)	$f(x)=\begin{cases} x\ln\lvert\sin x\rvert, & x\neq 0 \\ 0, & x=0\end{cases}$	(r)	Differentiable at $x = 0$
(D)	$f(x)=\begin{cases} xe^{1/x}, & x\neq 0 \\ 0, & x=0\end{cases}$	(s)	Non-differentiable at $x = 0$

68. Match the following.

	Column I		Column II
(A)	$f(x)=\sin(n[\pi])$ (where $[\cdot]$ denote G.I.F)	(p)	Differentiable everywhere
(B)	$f(x)=\sin((x-[x])\pi)$ (where $[\cdot]$ denote G.I.F)	(q)	Non-differentiable at $x = 2$
(C)	$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & \text{if } x\neq 0 \\ 0, & \text{if } x=0\end{cases}$	(r)	Non-differentiable at -1 and 1
(D)	$f(x)=\lvert 2-x\rvert+[2+x]$ (where $[\cdot]$ denote G.I.F)	(s)	Continuous at $x = 0$ but not differentiable at $x = 0$

Integer Answer Type

69. Let $f(x)=[x]+\left[x+\frac{1}{4}\right]+\left[x+\frac{1}{2}\right]+\left[x+\frac{3}{4}\right]$. Then no. of points of discontinuity of $f(x)$ in [0,1] is ($[\cdot]$ denotes G.I.F)

70. If $\alpha\in(-\infty,-1)\cup(-1,0)$ then the number of points where the function $f(x)=\left|x^2+(\alpha-1)\lvert x\rvert-\alpha\right|$ is not differentiable is

71. Let $f(x)=\begin{cases} x^2\sum_{r=0}^{\left[\frac{1}{\lvert x\rvert}\right]} r; & x\neq 0 \\ \frac{k}{2}; & \text{otherwise}\end{cases}$

($[\cdot]$ denotes the greatest integer function). The value of k such that f become continuous at $x = 0$ is

72. Let $f(x) = [x^2]\sin\pi x$, $x \in R$ the number of points in the interval (0, 3] at which the function is discontinuous is

73. If the function f defined by $f(x)=\dfrac{\log(1+x)^{1+x}}{x^2}-\dfrac{1}{x}$, $x \neq 0$ is continuous at $x = 0$, then $6\,(f(0)) =$

74. If $f(x)=\begin{cases}\dfrac{\tan[x^2]\pi}{ax^2}+ax^3+b &, 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x &, 1<x\le 2\end{cases}$ is differentiable in [0, 2], then $b=\dfrac{\pi}{k_1}-\dfrac{26}{k_2}$. Find $k_2 - k_1$ {where [·] denotes greatest integer function}.

SOLUTIONS

1. (b) : Here $f(0) = 0$

Since $-1\le \sin\dfrac{1}{x}\le 1 \Rightarrow -|x|\le x\sin\dfrac{1}{x}\le |x|$

We know that $\lim\limits_{x\to 0}|x|=0$ and $\lim\limits_{x\to 0}-|x|=0$

In this way $\lim\limits_{x\to 0}f(x)=0.$

2. (c)

3. (c) : $\lim\limits_{n\to\infty}\dfrac{n(2n+1)^2}{(n+2)\,(n^2+3n-1)}$

$$=\lim_{n\to\infty}\frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$$

$$=\lim_{n\to\infty}\frac{n^3\left(4+\frac{4}{n}+\frac{1}{n^2}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)}=4$$

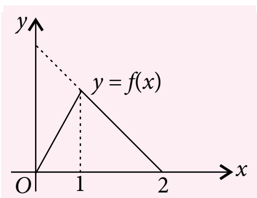
4. (b)

5. (a) : $\lim\limits_{x\to 1-}f(x)=\lim\limits_{h\to 0}(1-h)=1$

$\lim\limits_{x\to 1-}f(x)=\lim\limits_{h\to 0}(1-h)=1$

and $\lim\limits_{x\to 1+}f(x)=\lim\limits_{h\to 0}2-(1+h)=1$

Hence limit of function is 1.



6. (d)

7. (a) : $\lim\limits_{x\to 0}\dfrac{\log\cos x}{x}=\lim\limits_{x\to 0}\dfrac{\log\left[1-2\sin^2\frac{x}{2}\right]}{x}$

$$=\lim_{x\to 0}\frac{-\left[2\sin^2\frac{x}{2}+\left(\frac{2\sin^2\frac{x}{2}}{2}\right)^2+\ldots\ldots\right]}{x}=0$$

8. (b) : Applying L-Hospital's rule,

$$\lim_{x\to 9}\frac{\frac{1}{2\sqrt{f(x)}}\cdot f'(x)}{\frac{1}{2\sqrt{x}}}=\frac{\frac{f'(9)}{\sqrt{f(9)}}}{\frac{1}{\sqrt{9}}}=\frac{\frac{4}{3}}{\frac{1}{3}}=4$$

9. (a) : $\lim\limits_{x\to 0}\dfrac{e^{\sin x}-1}{x}=\lim\limits_{x\to 0}\dfrac{e^{\sin x}-1}{\sin x}\times\dfrac{\sin x}{x}$

$$=\lim_{x\to 0}\frac{e^{\sin x}-1}{\sin x}\times\lim_{x\to 0}\frac{\sin x}{x}=1\times 1=1.$$

10. (a)

11. (a) : $\lim\limits_{x\to\frac{\pi}{2}}\tan x\log\sin x=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\log\sin x}{\cot x}$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{\sin x}\cos x}{-\text{cosec}^2 x}=0 \text{ (Applying L-Hospital's rule)}$$

12. (b) : $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{a+b}{2}\right)x\cdot\sin\left(\frac{b-a}{2}\right)x}{\left(\frac{a+b}{2}\right)x\cdot\frac{2}{a+b}\cdot\frac{2}{b-a}\cdot\left(\frac{b-a}{2}\right)x}=\frac{b^2-a^2}{2}$$

13. (c) : $\lim\limits_{x\to 0}\dfrac{x[{}^5C_1+{}^5C_2x+{}^5C_3x^2+{}^5C_4x^3+{}^5C_5x^4]}{x[{}^3C_1+{}^3C_2x+{}^3C_3x^2]}=\dfrac{5}{3}.$

14. (d) : $\lim\limits_{x\to 0}\dfrac{2\sin 4x\cos 2x}{2\sin x\cos 4x}$

$$=\lim_{x\to 0}4\left(\frac{\sin 4x}{4x}\right)\left(\frac{x}{\sin x}\right)\frac{\cos 2x}{\cos 4x}=4$$

15. (b) : $\lim\limits_{x\to 0}x\log\sin x=\lim\limits_{x\to 0}\log(\sin x)^x$

$$=\log\left[\lim_{x\to 0}(1+\sin x-1)^{\frac{x(\sin x-1)}{\sin x-1}}\right]$$

$$=\log_e[e^{\lim_{x\to 0}x(\sin x-1)}]=\log_e 1.$$

16. (a)

17. (c) : $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

$=\lim_{h\to 0}\frac{2(a+h)\sin(a+h)+(a+h)^2\cos(a+h)}{1}$

$= 2a\sin a + a^2\cos a.$

18. (b) **19. (a)** **20. (a)**

21. (c) : Put $\cos^{-1}x = y$ and $x \to 1 \Rightarrow y \to 0$

$\therefore \lim_{x\to 1}\frac{1-\sqrt{x}}{(\cos^{-1}x)^2}=\lim_{y\to 0}\frac{1-\sqrt{\cos y}}{y^2}$

Now rationalizing it, we get $\lim_{y\to 0}\frac{(1-\cos y)}{y^2(1+\sqrt{\cos y})}$

$=\lim_{y\to 0}\frac{1-\cos y}{y^2}\cdot\lim_{y\to 0}\frac{1}{1+\sqrt{\cos y}}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}.$

22. (b) : $\lim_{x\to\pi/4}\frac{\sqrt{2}\cos x-1}{\cot x-1}$

Apply L'Hospitals's rule, we get

$\lim_{x\to\pi/4}\sqrt{2}\sin^3 x=\frac{\sqrt{2}}{2\sqrt{2}}=\frac{1}{2}.$

23. (b) : Let $\tan^{-1}2x=\theta \Rightarrow x=\frac{1}{2}\tan\theta$ and as $x \to 0$, $\theta \to 0$

$\Rightarrow \lim_{x\to 0}\frac{x}{\tan^{-1}2x}=\lim_{\theta\to 0}\frac{\frac{1}{2}\tan\theta}{\theta}=\frac{1}{2}.$

24. (d) : $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}=\lim_{x\to 0}\frac{|\sin x|}{x}$

So, $\lim_{x\to 0^+}\frac{|\sin x|}{x}=1$ and $\lim_{x\to 0^-}\frac{|\sin x|}{x}=-1$

Hence limit does not exist.

25. (b) : Let $A=\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x+3}$

$=\lim_{x\to\infty}\left(1+\frac{1}{x+1}\right)^{x+3}=\lim_{x\to\infty}\left[\left(1+\frac{1}{x+1}\right)^{x+1}\right]^{\frac{(x+3)}{(x+1)}}=e$

26. (c) : Apply L-Hospital's rule, we get

$\lim_{x\to 0}\frac{\cos x-\frac{1}{1-x}}{2x}=\lim_{x\to 0}\frac{-\sin x-\frac{1}{(1-x)^2}}{2}=-\frac{1}{2}$

27. (a) : Putting $x=\frac{1}{t}$, the given limit

$=\lim_{t\to 0}\frac{\frac{\sin t}{t}-1}{t-1}=\frac{1-1}{0-1}=0,$

28. (b)

29. (b) : $\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$

$=\lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$

$=\lim_{x\to\infty}\frac{\sqrt{1+x^{-1/2}}}{\sqrt{1+\sqrt{x^{-1}+x^{-3/2}}}+1}=\frac{1}{2}$

30. (c) : $f(x)=\left\{\frac{x^2-4x+3}{x^2-1}\right\}$, for $x\neq 1$ and $f(x)=2$, for $x = 1$

$\therefore f(1)=2,\ f(1^+)=\lim_{x\to 1^+}\frac{x^2-4x+3}{x^2-1}=\lim_{x\to 1^+}\frac{(x-3)}{(x+1)}=-1$

$f(1^-)=\lim_{x\to 1^-}\frac{x^2-4x+3}{x^2-1}=-1$

Hence the function is discontinuous at $x = 1$.

31. (c)

32. (b) : When $0 \le x < 1$

$f(x)$ doesn't exist as $[x] = 0$ here.

Also, $\lim_{x\to 1^+}f(x)$ and $\lim_{x\to 1^-}f(x)$ does not exist.

Hence $f(x)$ is discontinuous at all integers and also in $(0, 1)$.

33. (b) : $\lim_{x\to 0^-}f(x)=0,\ f(0)=0,\ \lim_{x\to 0^+}f(x)=-4$

$\therefore$ $f(x)$ is discontinuous at $x = 0$

and $\lim_{x\to 1^-}f(x)=1$ and $\lim_{x\to 1^+}f(x)=1,\ f(1)=1$

Hence $f(x)$ is continuous at $x = 1$

Also, $\lim_{x\to 2^-}f(x)=4(2)^2-3\cdot 2=10$

$f(2)=10$ and $\lim_{x\to 2^+}f(x)=3(2)+4=10$

Hence $f(x)$ is continuous at $x = 2$.

34. (d): $\lim\limits_{x\to 4^-} f(x)=\lim\limits_{h\to 0} f(4-h)$

$$=\lim_{h\to 0}\frac{4-h-4}{|4-h-4|}+a=\lim_{h\to 0}-\frac{h}{h}+a=a-1$$

$$=\lim_{x\to 4^+} f(x)=\lim_{h\to 0} f(4+h)=\lim_{h\to 0}\frac{4+h-4}{|4+h-4|}+b=b+1$$

and $f(4) = a + b$

Since $f(x)$ is continuous at $x = 4$

Therefore $\lim\limits_{x\to 4-} f(x)=f(4)=\lim\limits_{x\to 4+} f(x)$

$\Rightarrow\ a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1$.

35. (c) **36. (c)**

37. (c) : For continuity at all $x \in R$, we must have

$$f\left(-\frac{\pi}{2}\right)=\lim_{x\to(-\pi/2)^-}(-2\sin x)=\lim_{x\to(-\pi/2)^+}(A\sin x+B)$$

$\Rightarrow\ 2 = -A + B$ ....(i)

and $f\left(\frac{\pi}{2}\right)=\lim\limits_{x\to(\pi/2)^-}(A\sin x+B)=\lim\limits_{x\to(\pi/2)^+}(\cos x)$

$\Rightarrow\ 0 = A + B$ ....(ii)

From (i) and (ii) we get, $A = -1$ and $B = 1$.

38. (b)

39. (a) : $f(x)=x^p\sin\frac{1}{x}, x\neq 0$ and $f(x)=0, x=0$

Since at $x = 0$, $f(x)$ is a continuous function

$\therefore\ \lim\limits_{x\to 0} f(x)=f(0)=0 \Rightarrow \lim\limits_{x\to 0} x^p\sin\frac{1}{x}=0\Rightarrow p>0$

$f(x)$ is differentiable at $x = 0$, if $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}$ exists

$\Rightarrow\ \lim\limits_{x\to 0}\frac{x^p\sin\frac{1}{x}-0}{x-0}$ exists $\Rightarrow\ \lim\limits_{x\to 0} x^{p-1}\sin\frac{1}{x}$ exists

$\Rightarrow\ p - 1 > 0$ or $p > 1$

If $p \le 1$, then $\lim\limits_{x\to 0} x^{p-1}\sin\left(\frac{1}{x}\right)$ does not exist and at $x = 0$ $f(x)$ is not differentiable.

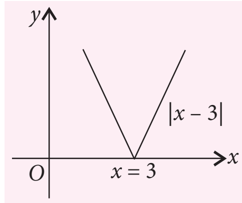
$\therefore$ for $0 < p \le 1$ $f(x)$ is a continuous function at $x = 0$ but not differentiable.

40. (b)

41. (d): $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{h\to 0} f(3-h)=\lim\limits_{h\to 0}|3-h-3|=0$

$\lim\limits_{x\to 3^+} f(x)=\lim\limits_{h\to 0} f(3+h)$

$=\lim\limits_{h\to 0}|3+h-3|=0$



$\because\ \lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x)=f(3)$

Hence f is continuous at $x = 3$

Now $L f'(3)=\lim\limits_{h\to 0}\frac{f(3-h)-f(3)}{-h}$

$$=\lim_{h\to 0}\frac{|3-h-3|-0}{-h}=\lim_{h\to 0}\frac{h}{-h}=-1$$

$$R f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{|3+h-3|-0}{h}=1$$

$\because\ Lf'(3) \neq Rf'(3)$. Hence f is not differentiable at $x = 3$.

42. (b) **43. (c)**

44. (a) : $f'(k^-)=\lim\limits_{h\to 0}\frac{[k-h]\sin\pi(k-h)-[k]\sin\pi k}{-h}$

$$=\lim_{h\to 0}\frac{(-1)^{k-1}(k-1)\sin\pi h-k\times 0}{-h}=(-1)^k\cdot(k-1)\pi$$

45. (b): $L f'(1)=\lim\limits_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$$=\lim_{h\to 0}\frac{m(1-h)^2-m}{-h}=\lim_{h\to 0}\frac{m[1+h^2-2h-1]}{-h}$$

$$=\lim_{h\to 0} m(2-h)=2m$$

and $R f'(1)=\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim\limits_{h\to 0}\frac{2(1+h)-m}{h}=2$

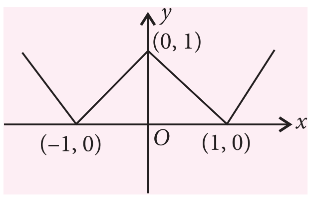
For differentiability, $Lf'(1) = Rf'(1)$.

Hence $m = 1$

46. (b) : $=\begin{cases}|x|-1, & |x|-1\ge 0\\ -|x|+1, & |x|-1<0\end{cases}$

$$=\begin{cases}|x|-1, & x\le -1 \text{ or } x\ge 1\\ -|x|+1, & -1<x<1\end{cases}$$

$$=\begin{cases}-x-1, & x\le -1\\ x+1, & -1<x<0\\ -x+1, & 0\le x<1\\ x-1, & x\ge 1\end{cases}$$



From the graph. It is clear that $f(x)$ is not differentiable at $x = -1, 0$ and 1.

47. (a, b) : If $f(x)$ is continuous at $x = a$, $\lim\limits_{x\to a} f(x) = f(a)$

Let $a \in R$ then $\lim\limits_{x\to a} f(x) = \lim\limits_{h\to 0} f(a+h)$

$= \lim\limits_{h\to 0}(f(a)+f(h)) = f(a) + \lim\limits_{x\to 0} f(h)$

$= f(a) + f(0) = f(a + 0) = f(a)$

$\Rightarrow$ 'f' is continuous $\forall x \in R$, as 'a' is arbitrary

$\because$ $f(x + y) = f(x) + f(y) \Rightarrow f(0) = 0\, f(1)$

For any +ve inteteger 'n'

$f(1) = f(1 + 1 + ... + n \text{ times}) = n\, f(1)$

For any –ve integer 'm' we have

$0 = f(0) = f[m + (-m)] = f(m) + f(-m)$

$\Rightarrow$ $f(m) = -f(-m) = -(-m)\, f(1) = m\, f(1)$

Let p/q be any rational number where 'q' is a +ve integer and p is any integer, +ve, –ve or zero.

Then $f\left(q.\frac{p}{q}\right) = f\left(\frac{p}{q}+\frac{p}{q}+\text{..........}q \text{ times}\right)$

$= f\left(\frac{p}{q}\right) + f\left(\frac{p}{q}\right) + \text{........}q \text{ times} = q.f\left(\frac{p}{q}\right)$

$\Rightarrow$ $f(p) = q \cdot f(p/q)$

But $f(p) = p \cdot f(1)$ from previous cases.

$\therefore$ $p \cdot f(1) = q \cdot f(p/q)$ $\Rightarrow$ $f(p/q) = \frac{p}{q}.f(1)$

48. (a, b, c)

49. (a, b) : Clearly $\phi'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ if $x \neq 0$ and 0 if $x = 0$.

$\Rightarrow$ $\phi'(x)$ is discontinuous at $x = 0$, as $\cos\left(\frac{1}{x}\right)$ is oscillating in the neighbour hood of '0'.

50. (a, c) : $|BD - CD| = \left|r\left(\cot\frac{B}{2} - \cot\frac{C}{2}\right)\right|$

$= r\left|\dfrac{\sin\left(\frac{B-C}{2}\right)}{\sin\frac{B}{2}\sin\frac{C}{2}}\right| = \left|4R\sin\frac{A}{2}\sin\frac{B-C}{2}\right|$

$= \left|4R\cos\frac{B+C}{2}\sin\frac{B-C}{2}\right| = |2R(\sin B - \sin C)| = |b - c|$

51. (b, c) : $f'(0) = \lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x} = \lim\limits_{x\to 0}\dfrac{1}{1+e^{1/x}}$

$f'(0^+) = \lim\limits_{x\to 0^+}\dfrac{1}{1+e^{1/x}} = 0$

$f'(0^-) = \lim\limits_{x\to 0^-}\dfrac{1}{1+e^{1/x}} = 1$

52. (a, d) : f is continuous in its domain $[-1, 1]$

$f'(x) = \dfrac{x}{2\sqrt{1-\sqrt{1-x^2}}\sqrt{1-x^2}}, x \neq 0, x \neq \pm 1$

53. (a, b, d) : $f(x) = \begin{cases}(x-a)\phi(x) \text{ if } x \geq a \\ (a-x)\phi(x) \text{ if } x < a\end{cases}$

$\therefore$ $f'(a^+) = \lim\limits_{x\to a}(x-a)\phi'(x) + \phi(x) = \phi(a)$

$f'(a^-) = \lim\limits_{x\to a}(a-x)\phi'(x) - \phi(x) = -\phi(a)$

54. (a, b, c) : If $|x| < 1$ and $|x| \neq \frac{1}{n}$,

then $\dfrac{1}{|x|} - 1 < \left[\dfrac{1}{|x|}\right] < \dfrac{1}{|x|}$

$\Rightarrow$ $1 - |x| < |x|\left[\dfrac{1}{|x|}\right] < 1 \Rightarrow f(x) = 0$

If $|x| > 1$, then $0 < \dfrac{1}{|x|} < 1$ and hence $\left(\dfrac{1}{|\lambda|}\right) = 0.$

Then $f(x) = 0$

Hence $f(x) = 0$ for all $x \in R$

55. (b, d) : $f(x) = 2 - \sin^{-1}x$ if $-1 \leq x \leq 0$

$2 + \sin^{-1}x$ if $0 < x \leq 1$

Hence f is continuous everywhere on the domain

$\dfrac{-1}{\sqrt{1-x^2}}$ if $-1 < x < 0$, $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ if $0 < x < 1$

$\therefore$ f is not differentiable at $x = 0$

56. (a, c, d) : $f'(x) = \begin{cases}-\cos\pi x, & \text{if } -1 < x < 0 \\ 1, & \text{if } x = 0 \\ \cos\pi x, & \text{if } 0 < x < 1\end{cases}$

$\therefore$ f is not continuous at $x = 0$

57. (d) : Degree of $p(x)$ is 2

58. (a) : $\dfrac{p'(x)}{|x|} = \dfrac{2ax+b}{|x|}$ is discontinuous at $x = 0$

$\because$ $\dfrac{2ax}{|x|}$ is discontinuous. We know if f is continuous and 'g' is discontinuous then $f + g$ is discontinuous.

59. (b) : $p(1) = 3$, $p(-1) = 5 \Rightarrow a = 4$, $b = -1$,

$\therefore$ $p(x) = 4x^2 - x$

$p'(x) = 8x - 1$, $\because$ g' is inverse of $p'(x)$

$g(x) = \dfrac{x+1}{8}; g'(0) = \dfrac{1}{8}$

60. (c) : Let $0 \le x < 1$, then

$$f(x) = \log(2+x) \quad \left(\because \lim_{n\to\infty} x^{2^n} = 0\right)$$

61. (b) : Let $x = 1, f(x) = \frac{1}{2}(\log 3 + \sin 1)$

62 (a) : If $x > 1$, then

$$\lim_{n\to\infty} \frac{\log(2+x) + x^{2^n} \sin x}{1 + x^{2^n}} = \lim_{n\to\infty} \frac{\left[\frac{\log(2+x)}{x^{2^n}} + \sin x\right]}{\frac{1}{x^{2^n}} + 1}$$

$$\therefore \; f(x) = \begin{cases} \log(2+x) & \text{, if } 0 < x < 1 \\ \frac{1}{2}(\log 3 + \sin 1) & \text{, if } x = 1 \\ \sin x & \text{, if } x > 1 \end{cases}$$

$$= \sin x \left(\because \lim_{n\to\infty} \frac{1}{x^{2^n}} = 0\right)$$

63. (d) : $f(x) = \lim_{m\to\infty} (\sin^2 x)^m = 1, x = (2n+1)\frac{\pi}{2}, n \in I$

$= 0, x \ne (2n+1)\frac{\pi}{2}, n \in I$

$\therefore$ $f(x)$ is discontinuous at $x = (2n+1)\frac{\pi}{2}, n \in I$

64. (c) : L.H.L.≠R.H.L., as $\lim_{x\to 1^-} -\sqrt{2}\left(\frac{\sin(x-1)}{x-1}\right) = -\sqrt{2}$

and $\lim_{x\to 1^+} \sqrt{2}\left(\frac{\sin(x-1)}{x-1}\right) = \sqrt{2}$

65. (b) : $f(x) = \lim_{x\to 0}(1+x)^{\cot x} = e^{\lim_{x\to 0} \cot x(1+x-1)}$

$= e^{\lim_{x\to 0} \frac{x}{\tan x}} = e$

66. (A)→(p, q), (B)→(p, q), (C)→(r, s, p), (D)→(r, s, p)

(A) $y = \begin{cases} \pi - 2\tan^{-1} x, & x > 1 \\ 2\tan^{-1} x & -1 \le x \le 1 \\ -\pi - 2\tan^{-1} x & x < -1 \end{cases}$

$$\therefore \frac{dy}{dx} = \begin{cases} \frac{-2}{1+x^2}, & |x| > 1 \\ \frac{2}{1+x^2}, & |x| < 1 \\ \text{does not exist,} & \text{at } |x| = 1 \end{cases}$$

Hence, $f(x)$ is not differentiable at $x = -1, 1$

(B) $y = \begin{cases} \pi + 2\tan^{-1} x, & \pi < -1 \\ 2\tan^{-1} x, & -1 < x < 1 \\ -\pi + 2\tan^{-1} x, & x > 1 \end{cases}$

$$\therefore \frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2}, & x \in R - \{-1, 1\} \\ \text{does not exist,} & \text{at } x = \{-1, 1\} \end{cases}$$

Hence, $f(x)$ is not differentiable at $x = -1, 1$

(C) $f(x) = \begin{cases} 2\pi + 3\cos^{-1} x, & -1 \le x \le -\frac{1}{2} \\ 2\pi - 3\cos^{-1} x, & -\frac{1}{2} \le x \le \frac{1}{2} \\ 3\cos^{-1} x, & \frac{1}{2} \le x \le 1 \end{cases}$

$$\therefore \frac{dy}{dx} = f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}}, & \frac{1}{2} < |x| < 1 \\ \text{does not exist,} & |x| = \frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}}, & |x| < \frac{1}{2} \end{cases}$$

Hence, $f(x)$ is not differentiable at $x = \frac{-1}{2}, \frac{1}{2}$

(D) $f(x) = \begin{cases} \pi - 3\sin^{-1} x, & \frac{1}{2} \le x \le 1 \\ 3\sin^{-1} x, & -\frac{1}{2} \le x \le \frac{1}{2} \\ -\pi - 3\sin^{-1} x, & -1 \le x \le -\frac{1}{2} \end{cases}$

$$f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}}, & \frac{1}{2} < |x| < 1 \\ \text{does not exist,} & |x| = \frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}}, & |x| < \frac{1}{2} \end{cases}$$

Hence, $f(x)$ is not differentiable at $x = \frac{-1}{2}, \frac{1}{2}$.

67. (A)→(p, s), (B)→(p, r), (C)→(p, s), (D)→(q, s)

(A) $f(x) = \begin{cases} -x, x < 0 \\ x, x \ge 0 \end{cases}$

continuous but not differentiable at $x = 0$

(B) $f(x) = x^n|x|$

$\Rightarrow$ L.H.D = R.H.D = 0 at $x = 0$

(C) L.H.L = R.H.L = $f(0)$ = 0 but L.H.D and R.H.D are not finite.

(D) $\text{L.H.L.}=0, \text{R.H.L.}=\lim\limits_{x\to 0}\dfrac{e^{1/x}}{1/x}$

$$=\lim_{x\to 0}\frac{e^{1/x}(-1/x^2)}{(-1/x^2)}=\lim_{x\to 0}e^{1/x}=\infty$$

68. (A) $\to$ (p), (B) $\to$ (q, r, s), (C) $\to$ (s), (D) $\to$ (q, r)

(A) We know that $[x]\in I, \forall\, x\in R$

$\therefore$ $\sin(\pi[x])=\sin \pi x=0\ \ \forall\, x\in R$

Since every constant function is differentiable in its domain.

$\therefore$ $\sin(\pi[x])$ is differentiable everywhere.

(B) $f(x) = \sin[(x - [x])\pi]$

Since $x - [x]$ is not differentiable at integral points

$\therefore$ $f(x) = \sin(\pi(x - [x]))$ is not differentiable at $x \in I$

$\therefore$ It is not differentiable at -1, 1

(C) $\lim\limits_{x\to 0} f(x)=0$ (a finite quantity between -1 and 1)

$= 0 = f(0)$

$\therefore$ $f(x)$ is continuous at $x = 0$ and

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

Which does not exists.

$\therefore$ $f(x)$ is not differentiable at $x = 0$.

(D) $|2 - x|$ is continuous everywhere and $[2 + x]$ is discontinuous at all integral values of x.

$\therefore$ $f(x)$ is discontinuous at $x = 2$.

$\therefore$ $f(x)$ is not differentiable at $x = 2$.

69. (4) : $[x]+\left[x+\dfrac{1}{4}\right]+\left[x+\dfrac{2}{4}\right]+\left[x+\dfrac{3}{4}\right]=[4x]$

$\therefore$ $f(x) = [4x]$ which will become discontinuous at $x=\dfrac{1}{4},\dfrac{2}{4},\dfrac{3}{4},\dfrac{4}{4}$

70. (5) : Given $f(x)=\left|x^2+(\alpha-1)|x|-\alpha\right|$

$\Rightarrow$ $f(x) = |(|x| - 1)(|x| + \alpha)|$

$\therefore$ $f(x)$ is not differentiable at '5' points.

71. (1) : In the vicinity of $x = 0$,we have

$$x^2\sum_{r=0}^{\left[\frac{1}{|x|}\right]} r = x^2\left(1+2+3+\ldots+\left[\frac{1}{|x|}\right]\right)$$

Use sandwich theorem

$$P=x^2\left(1+2+3+\ldots+\left[\frac{1}{|x|}\right]\right)=\frac{x^2\left(1+\left[\frac{1}{|x|}\right]\right)}{2}\left[\frac{1}{|x|}\right]$$

So, $\dfrac{1}{2}(1-|x|)<P\le\dfrac{1}{2}(1+|x|)$

Then the limit is 1/2.

72. (6) : $f(x)=\begin{cases}0 & 0<x<1\\ \sin\pi x & 1\le x<\sqrt{2}\\ 2\sin\pi x & \sqrt{2}\le x<\sqrt{3}\\ 3\sin\pi x & \sqrt{3}\le x<2\\ 4\sin\pi x & 2\le x<\sqrt{5}\ \textit{etc.}\end{cases}$

The function is discontinuous at $x=\sqrt{2},\sqrt{3},\sqrt{5},\ldots.\sqrt{K}$ where K is not a perfect square.

$\therefore$ Points of discontinuity are 6.

73. (3) : Since $f(x)$ is continuous at $x = 0$

$\therefore$ $f(0)=\lim\limits_{x\to 0}\dfrac{\log(1+x)^{1+x}-x}{x^2}$

$$=\lim_{x\to 0}\frac{(1+x)\log(1+x)-x}{x^2}=\lim_{x\to 0}\frac{1+\log(1+x)-1}{2x}=\frac{1}{2}$$

$\therefore$ $6f(0) = 3$

74. (8) : $f(x)=\begin{cases}ax^3+b & ,\ 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x & ,\ 1<x\le 2\end{cases}$

$$f'(x)=\begin{cases}3ax^2 & ,\ 0<x<1\\ -2\pi\sin\pi x+\dfrac{1}{1+x^2} & ,\ 1<x\le 2\end{cases}$$

As the function is differentiable in [0, 2]

$\Rightarrow$ function is differentiable at $x = 1$

$\therefore$ $f'(1^-)=f'(1^+)\ \Rightarrow\ 3a=\dfrac{1}{2}\ \Rightarrow\ a=\dfrac{1}{6}$

Function will also be continuous at $x = 1$

$\therefore$ $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^+}f(x)\ \Rightarrow\ a+b=-2+\dfrac{\pi}{4}$

$\therefore$ $b=-2-\dfrac{1}{6}+\dfrac{\pi}{4}=\dfrac{\pi}{4}-\dfrac{13}{6}\ \Rightarrow\ k_1=4\ \&\ k_2=12$

$\Rightarrow$ $k_2 - k_1 = 8$

CLASS XII | **Series 6**

ACE YOUR WAY CBSE

Differential Equations

IMPORTANT FORMULAE

- An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation.
- Order of a differential equation is the order of the highest order derivative occurring in the differential equation.
- Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.
- The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called particular solution.
- $y_1 = y' = \frac{dy}{dx}, y_2 = y'' = \frac{d^2y}{dx^2}, y_3 = y''' = \frac{d^3y}{dx^3}, \ldots$ $\ldots, y_n = \frac{d^ny}{dx^n}$
- A differential equation is linear if it can be expressed as
  $$a_0\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_{n-1}\frac{dy}{dx} + a_ny = P_n$$
  Where $a_0, a_1, a_2, \ldots\ldots, a^n$ and p_n are constants or functions of independent variable x.
- If the equation is $\frac{dy}{dx} = f(x)$, then $y = \int f(x)dx + C$.
- **Variable Separable :** If the equation is of the form $\frac{dy}{dx} = f(x)\cdot g(y)$, then $\int \frac{dy}{g(y)} = \int f(x)dx + C$.
- **Reducible to Variable separable :** If the equation is of the form $\frac{dy}{dx} = f(ax+by+c)$, then put $ax + by + c = z$.
- **Homogeneous Equation :** If the equation is of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$.
- **Linear Equation :** If the equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then solution of differential equation is $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C$ where $e^{\int Pdx}$ is the integrating factor (I.F.).

  Or

  If the equation is of the form $\frac{dx}{dy} + Px = Q$, where P, Q are functions of y, then $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + C$ where $e^{\int Pdy}$ is the integrating factor.

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Find the differential equation corresponding to $y = Ae^x + Be^{-x}$.

2. Solve: $\frac{dy}{dx} = (e^x + 1)y$

3. Find the I.F of $(1 - x^2)\frac{dy}{dx} + xy = ax$.

4. Find the order and degree of $y = px + \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$.

5. Find the differential equation of all non-vertical lines in a plane.

SHORT ANSWER TYPE

6. Show that the function $y = Ax + \frac{B}{x}$ is a solution of the differential equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

7. Find the particular solution of $y' = y\cot 2x$, $y\left(\frac{\pi}{4}\right) = 2$

8. The slope of the tangent at a point $P(x, y)$ on a curve is $(-x/y)$. If the curve passes through the point $(3, -4)$, find the equation of the curve.

9. Solve : $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

10. Show that $y = x^2 + 2x + 1$ is the solution of the initial value problem $\frac{d^3y}{dx^3} = 0, y(0) = 1, y'(0) = 2, y''(0) = 2.$

LONG ANSWER TYPE - I

11. Form the differential equation corresponding to $(x - a)^2 + (y - b)^2 = r^2$ by eliminating a and b.

12. Solve : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$

13. Solve the following differential equations :

(i) $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9, x \neq -2$

(ii) $\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$

14. Find the equation of the curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

15. Solve $(x + y)^2\frac{dy}{dx} = a^2$

LONG ANSWER TYPE - II

16. Form the differential equation of the family of circles in the first quadrant which touch the coordinates axes.

17. If the tangent at any point P of a curve meets the axis of X in T. Find the curve for which $OP = PT$, O being the origin.

18. Find the particular solution of $(x+y)dy+(x-y)dx=0$, given that $y = 1$ when $x = 1$.

19. Solve: $\frac{dy}{dx} + y = \cos x - \sin x$, $y(0) = 1$.

20. Experiments show that the rate of inversion of cane sugar in dilute solution is proportional to the concentration $y(t)$ of the unaltered sugar. Suppose that the concentration is $1/100$ at $t = 0$ and is $1/300$ at $t = 10$ hours. Find $y(t)$.

SOLUTIONS

1. We have, $y = Ae^x + Be^{-x}$ ...(i)

Differentiating (i) w.r.t. x, we get $\frac{dy}{dx} = Ae^x - Be^{-x}$

Differentiating again w.r.t. x, we get

$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} = y$ is the required differential equation.

2. We have, $\frac{dy}{dx} = (e^x + 1)y \Rightarrow \frac{dy}{y} = (e^x + 1)dx$

On Integrating both sides, we get $\int\frac{dy}{y} = \int(e^x + 1)dx$

$\Rightarrow \log y = e^x + x + c.$

3. We have, $(1 - x^2)\frac{dy}{dx} + xy = ax$

$$\Rightarrow \frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{ax}{1-x^2}$$

$$\therefore \text{I.F.} = e^{\int\frac{x}{1-x^2}dx} = e^{-\frac{1}{2}\int\frac{-2x}{1-x^2}dx}$$

$$= e^{-\frac{1}{2}\log(1-x^2)} = e^{\log(1-x^2)^{-1/2}}$$

$$= (1 - x^2)^{-1/2} = \frac{1}{\sqrt{1-x^2}}$$

4. Given, $y - px = \sqrt{a^2p^2 + b^2}$

$\Rightarrow (y - px)^2 = a^2p^2 + b^2$

$\Rightarrow (x^2 - a^2)p^2 - 2xyp + (y^2 - b^2) = 0$

$$\Rightarrow (x^2 - a^2)\left(\frac{dy}{dx}\right)^2 - 2xy\left(\frac{dy}{dx}\right) + (y^2 - b^2) = 0$$

Hence, order 1 and degree 2.

5. The equation of the family of non-vertical lines in plane is $ax + by = 1$, where $b \neq 0$ and $a \in R$.
Differentiating w.r.t. x, we get

$$a + b\frac{dy}{dx} = 0$$

On differentiating again w.r.t. x, we get

$b\dfrac{d^2y}{dx^2} = 0 \Rightarrow \dfrac{d^2y}{dx^2} = 0$ is the required differential equation.

6. The given function is : $y = Ax + \dfrac{B}{x}$ ...(i)

Differentiating w.r.t. x, we get $\dfrac{dy}{dx} = A - \dfrac{B}{x^2}$ ...(ii)

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -B(-2x^{-3}) = \frac{2B}{x^3} \Rightarrow x^2\frac{d^2y}{dx^2} = \frac{2B}{x} \quad \text{...(iii)}$$

Now, $x\dfrac{dy}{dx} - y = x\left(A - \dfrac{B}{x^2}\right) - \left(Ax + \dfrac{B}{x}\right)$

[Using (i) and (ii)]

$$\Rightarrow x\frac{dy}{dx} - y = Ax - \frac{B}{x} - Ax - \frac{B}{x} = -\frac{2B}{x} \quad \text{...(iv)}$$

Adding (iii) and (iv), we get

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{2B}{x} - \frac{2B}{x} = 0$$

Hence, $y = Ax + \dfrac{B}{x}$ is a solution of above differential equation.

7. We have, $y' = y \cot 2x \Rightarrow \dfrac{dy}{dx} = y \cot 2x$

$$\Rightarrow \frac{dy}{y} = \cot 2x \, dx$$

Integrating both sides, we get

$$\int \frac{dy}{y} = \int \cot 2x \, dx$$

$$\Rightarrow \log|y| = \frac{1}{2}\log|\sin 2x| + \log c$$

$$\Rightarrow \log|y| = \log\left|\sqrt{\sin 2x}\right| + \log c$$

$$\Rightarrow \log|y| = \log\left|c\sqrt{\sin 2x}\right| \Rightarrow y = c\sqrt{\sin 2x}$$

Given $y\left(\dfrac{\pi}{4}\right) = 2$

$$\Rightarrow 2 = c\sqrt{\sin\frac{\pi}{2}} \Rightarrow c = 2 \;\therefore\; y = 2\sqrt{\sin 2x}$$

8. Given : $\dfrac{dy}{dx} = -\dfrac{x}{y} \Rightarrow y\,dy + x\,dx = 0$

Integrating both sides, we get

$$\frac{y^2}{2} + \frac{x^2}{2} = c' \Rightarrow y^2 + x^2 = 2c'$$

$\Rightarrow x^2 + y^2 = c$, where $c = 2c'$.
Since it passes through $(3, -4)$
$\therefore \; (3)^2 + (4)^2 = c \Rightarrow c = 25.$
Hence, the required equation of the curve is $x^2 + y^2 = 25$.

9. We have, $\dfrac{dy}{dx} = -\dfrac{x + y\cos x}{1 + \sin x}$

$$\Rightarrow \frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = -\frac{x}{1+\sin x} \quad \text{...(i)}$$

$$\therefore \; \text{I.F.} = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = (1+\sin x)$$

$$y(1+\sin x) = \int -x\,dx + c$$

[Using : $y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + c$]

$$\Rightarrow y(1+\sin x) = -\frac{x^2}{2} + c$$

$\Rightarrow y = \dfrac{2c - x^2}{2(1+\sin x)}$, which gives the required solution.

10. We have, $y = x^2 + 2x + 1$

$$\Rightarrow \frac{dy}{dx} = 2x + 2, \; \frac{d^2y}{dx^2} = 2 \text{ and } \frac{d^3y}{dx^3} = 0,$$

Also, $y(0) = 0 + 0 + 1 = 1$, $\left(\dfrac{dy}{dx}\right)_{x=0} = 2$

and $\left(\dfrac{d^2y}{dx^2}\right)_{x=0} = 2$

$\Rightarrow y(0) = 1, y'(0) = 2$ and $y''(0) = 2$.
Hence, $y = x^2 + 2x + 1$ is the solution of the initial value problem.

11. We have, $(x - a)^2 + (y - b)^2 = r^2$ ...(i)
Differentiating both sides w.r.t. to x, we have

$$\Rightarrow 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow (x-a) + (y-b)\frac{dy}{dx} = 0 \quad \text{...(ii)}$$

Again differentiating both sides w.r.t to x, we get

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \text{...(iii)}$$

Form (ii) and (iii), we get

$$y - b = -\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \text{ and } x - a = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$$

Now, susbstituting these in the given equation, we obtain

$$\left\{\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}\right\}^2+\left\{\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right\}^2=r^2$$

$\Rightarrow \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$ is the required differential equation.

12. We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{\sqrt{x}}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \quad \text{...(i)}$$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$ where $P=\frac{1}{\sqrt{x}}$ and $Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

$\therefore$ I.F. $= e^{\int P\,dx}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

$$\Rightarrow ye^{2\sqrt{x}}=\int e^{2\sqrt{x}}\cdot\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx+c$$

$$\Rightarrow ye^{2\sqrt{x}}=\int\frac{1}{\sqrt{x}}dx+c \Rightarrow ye^{2\sqrt{x}}=2\sqrt{x}+c$$

$\Rightarrow y=(2\sqrt{x}+c)e^{-2\sqrt{x}}$, which gives the required solution.

13. **(i)** We have, $(x+2)\frac{dy}{dx}=x^2+4x-9$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2+4x-9}{x+2} \quad [\because x\neq -2]$$

$$\Rightarrow dy=\left(\frac{x^2+4x-9}{x+2}\right)dx$$

Integrating both sides, we get

$$\int dy=\int\frac{x^2+4x-9}{x+2}dx \Rightarrow \int dy=\int\left(x+2-\frac{13}{x+2}\right)dx$$

$$\Rightarrow y=\frac{x^2}{2}+2x-13\log|x+2|+c$$

(ii) We have, $\frac{dy}{dx}=\sin^3 x\cos^2 x+xe^x$

$\Rightarrow dy=(\sin^3 x\cos^2 x+x\,e^x)dx$

Integrating both sides, we get

$$\int dy=\int(\sin^3 x\cos^2 x+x\,e^x)\,dx$$

$$\Rightarrow \int dy=\int\sin^3 x\cos^2 x\,dx+\int x\,e^x\,dx$$

$$\Rightarrow \int dy=\int\cos^2 x\,(1-\cos^2 x)\sin x\,dx+\int xe^x dx$$

$\Rightarrow y=-\int t^2(1-t^2)\,dt+\{x\,e^x-\int e^x\,dx\}$, where $t=\cos x$

$$\Rightarrow y=-\left\{\frac{t^3}{3}-\frac{t^5}{5}\right\}+(xe^x-e^x)+c$$

$$\Rightarrow y=-\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+x\,e^x-e^x+c, \forall x\in R$$

14. The slope of the tangent at any point (x, y) on the curve $y=f(x)$ is given by $\frac{dy}{dx}$.

Also, $\frac{dy}{dx}=(x-y)^2$ (Given) ...(i)

$\therefore$ Let $x-y=v \Rightarrow 1-\frac{dy}{dx}=\frac{dv}{dx}$. Substituting these values in (i), we get

$$1-\frac{dv}{dx}=v^2 \Rightarrow \frac{dv}{dx}=1-v^2 \Rightarrow \frac{1}{1-v^2}dv=dx$$

Integrating, we obtain

$$\frac{1}{2}\log\left|\frac{1+v}{1-v}\right|=x+c \quad \text{or} \quad \log\left|\frac{1+v}{1-v}\right|=2x+2c$$

or, $\log\left|\frac{1+x-y}{1-x+y}\right|=2x+2c$ ...(ii)

It is given that the curve passes through the origin. Therefore, $y=0$ when $x=0$. Putting $x=0$, $y=0$ in (ii), we obtain $c=0$.

Putting $c=0$ in (ii), we obtain

$$\log\left|\frac{1+x-y}{1-x+y}\right|=2x \quad \text{or} \quad \log\left|\frac{1+x-y}{1-x+y}\right|=e^{2x}$$

or, $(1+x-y)=\pm(1-x+y)\,e^{2x}$, which is the equation of the curve.

15. Let $x+y=v$. Then,

$$1+\frac{dy}{dx}=\frac{dv}{dx} \Rightarrow \frac{dy}{dx}=\frac{dv}{dx}-1$$

On substituting in given equation, we get

$$v^2\left(\frac{dv}{dx}-1\right)=a^2 \quad \Rightarrow \quad v^2\frac{dv}{dx}=a^2+v^2$$

$\Rightarrow v^2\,dv=(a^2+v^2)dx$

$$\Rightarrow \frac{v^2}{v^2+a^2}dv=dx \quad \text{[By separating the variables]}$$

$$\Rightarrow \left(1-\frac{a^2}{v^2+a^2}\right)dv=dx$$

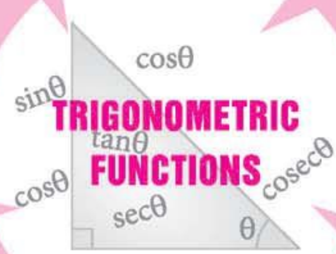
$$\Rightarrow \int 1\cdot dv-a^2\int\frac{1}{v^2+a^2}dv=\int dx+c$$

[On integration]

CONCEPT MAP

TRIGONOMETRIC FUNCTIONS

Class XI



Domain & Range

Function	Domain	Range
$\sin\theta$	R	$[-1, 1]$
$\cos\theta$	R	$[-1, 1]$
$\tan\theta$	$R-\left\{(2n+1)\frac{\pi}{2}, n\in Z\right\}$	R
$\sec\theta$	$R-\left\{(2n+1)\frac{\pi}{2}, n\in Z\right\}$	$R-(-1, 1)$
$\operatorname{cosec}\theta$	$R-\{n\pi, n\in Z\}$	$R-(-1, 1)$
$\cot\theta$	$R-\{n\pi, n\in Z\}$	R

Relation Between Degree and Radian

$$1 \text{ Radian} = \frac{180^\circ}{\pi} = 57^\circ 16' \text{ (approx.)}$$

$$1^\circ = \frac{\pi}{180}\text{ radian} = 0.01746 \text{ radian (approx.)}$$

Trigonometric Equations

- $\sin\theta = 0 \Rightarrow \theta = n\pi, n\in Z$
- $\cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2, n\in Z$
- $\tan\theta = 0 \Rightarrow \theta = n\pi, n\in Z$
- $\cot\theta = 0 \Rightarrow \theta = (2\pi+1)\pi/2, n\in Z$
- $\sin\theta = \sin\alpha, \theta = n\pi + (-1)^n\alpha, n\in Z$
- $\cos\theta = \cos\alpha, \theta = 2n\pi \pm \alpha, n\in Z$
- $\tan\theta = \tan\alpha, \theta = n\pi + \alpha, n\in Z$

Multiple Angles

- $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = 1 - \tan^2 A / 1 + \tan^2 A$
- $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

Solution of Triangles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{C}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

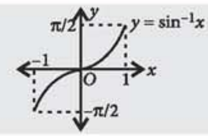
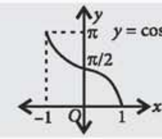
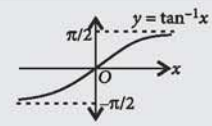
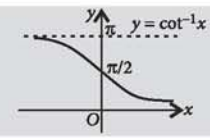
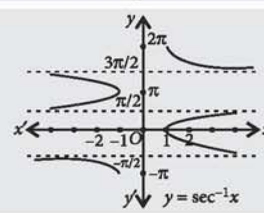
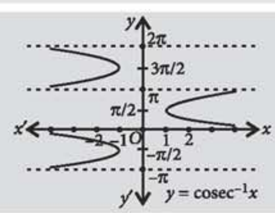
Sum & Difference of Two Angles

- $\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A\pm B) = \cos A\cos B \pm \sin A\sin B$
- $\tan(A\pm B) = \tan A \pm \tan B / 1 \mp \tan A\cdot\tan B$
- $\cot(A\pm B) = \cot A\cdot\cot B \mp 1/\cot B \pm \cot A$
- $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
- $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$
- $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
- $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

INVERSE TRIGONOMETRIC FUNCTIONS

CONCEPT MAP

Class XII

Function	Domain	Range	Graph
$y = \sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$	
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	
$y = \tan^{-1} x$	R	$(-\pi/2, \pi/2)$	
$y = \cot^{-1} x$	R	$(0, \pi)$	
$y = \text{cosec}^{-1} x$	$R - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$	
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \{\pi/2\}$	

Properties of Inverse Trigonometric Functions

1

$\sin(\sin^{-1} x) = x$ or $\sin^{-1}(\sin x) = x$

$\sin^{-1} 1/x = \text{cosec}^{-1} x, x \geq 1$ or $x \leq -1$

$\cos^{-1} 1/x = \sec^{-1} x, x \geq 1$ or $x \leq -1$

$\tan^{-1} 1/x = \cot^{-1} x, x > 0$

$\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1, 1]$

$\tan^{-1}(-x) = -\tan^{-1} x, x \in R$

$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x, |x| \geq 1$

$\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1, 1]$

$\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$

$\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \geq 1$

2

$\sin^{-1} x + \cos^{-1} x = \pi/2$

$\tan^{-1} x + \cot^{-1} x = \pi/2$

$\text{cosec}^{-1} x + \sec^{-1} x = \pi/2$

3

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1$$

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$$

4

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), -1 < x < 1$$

$$= \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \leq 1 = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0$$

$$\Rightarrow \quad v - a\tan^{-1}\left(\frac{v}{a}\right) = x + c$$

Hence, $(x + y) - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + c$, is the required solution.

16. Family of circles which touch the coordinates axes in first quadrant is: $(x - a)^2 + (y - a)^2 = a^2$ ...(i)
where a is the radius of the circles. Differentiating (i) w.r.t. x,

$$2(x-a)+2(y-a)\frac{dy}{dx}=0$$

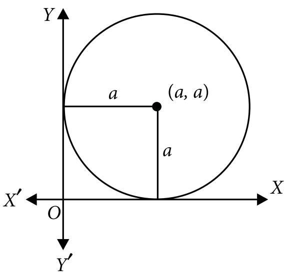
Putting $\frac{dy}{dx} = p$, we get

$(x - a) + (y - a)p = 0$

or $x - a + py - pa = 0$

$\Rightarrow \; x + py - a(1 + p) = 0$

$$\therefore \quad a = \frac{x+py}{1+p}$$



Putting value of a in (i), we get

$$\Rightarrow \left(x-\frac{x+py}{1+p}\right)^2+\left(y-\frac{x+py}{1+p}\right)^2=\left(\frac{x+py}{1+p}\right)^2$$

$$\Rightarrow \left[\frac{x(1+p)-x-py}{1+p}\right]^2+\left[\frac{y(1+p)-x-py}{1+p}\right]^2=\left(\frac{x+py}{1+p}\right)^2$$

$$\text{or,} \quad \left[\frac{(x-y)p}{1+p}\right]^2+\left[\frac{(y-x)}{1+p}\right]^2=\left[\frac{x+py}{1+p}\right]^2$$

Multiplying both sides by $(1 + p)^2$, we get

$$(x - y)^2 p^2 + (x - y)^2 = (x + py)^2$$

or, $(x - y)^2(1 + p^2) = (x + py)^2$

$$\Rightarrow \quad (x-y)^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=\left[x+y\frac{dy}{dx}\right]^2$$

This is the required differential equation.

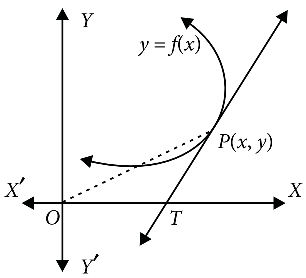
17. Let $y = f(x)$ be given curve and let $P(x, y)$ be a point on it. The equation of the tangent at P is

$$Y-y=\frac{dy}{dx}(X-x) \qquad \text{...(i)}$$

This meets X -axis at T. So, the x-coordinate of T is obtained by putting $Y = 0$ in (i).

Putting $Y = 0$ in (i), we get

$$0-y=\frac{dy}{dx}(X-x)$$



$$\Rightarrow X = x - y\frac{dx}{dy}$$

Thus, the coordinates of T are $\left(x-y\frac{dx}{dy},0\right)$

$$\therefore \quad PT=\sqrt{\left\{x-\left(x-y\frac{dx}{dy}\right)\right\}^2+(y-0)^2}$$

$$=\sqrt{y^2\left(\frac{dx}{dy}\right)^2+y^2} \text{ [Given]}$$

Now, $PT = OP$ (Given)

$$\Rightarrow \sqrt{y^2+y^2\left(\frac{dx}{dy}\right)^2}=\sqrt{x^2+y^2}$$

$$\Rightarrow y^2+y^2\left(\frac{dx}{dy}\right)^2=x^2+y^2 \Rightarrow y^2\left(\frac{dx}{dy}\right)^2=x^2$$

$$\Rightarrow y\frac{dx}{dy}=\pm x \Rightarrow \frac{dx}{x}=\pm\frac{dy}{y}$$

$\Rightarrow$ $\log x = \pm \log y + \log c$

$\Rightarrow$ $\log x = \log (c\,y)$ or, $\log x = \log\left(\frac{c}{y}\right)$

$\Rightarrow$ $x = c\,y$ or, $x = \frac{c}{y}$, are the equations of curve.

18. We have, $(x + y)dy + (x - y)dx = 0$

$\Rightarrow (x + y)dy = -(x - y)dx$

$$\Rightarrow \frac{dy}{dx}=\frac{y-x}{x+y} \qquad \text{...(i)}$$

$$\text{Put } y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx} \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$v+x\frac{dv}{dx}=\frac{(v-1)x}{(v+1)x} \Rightarrow x\frac{dv}{dx}=\frac{v-1}{v+1}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{v-1-v^2-v}{v+1} \Rightarrow \frac{v+1}{1+v^2}dv=-\frac{dx}{x}$$

$$\Rightarrow \int\frac{1}{1+v^2}dv+\int\frac{v}{1+v^2}dv=-\int\frac{dx}{x}$$

$$\Rightarrow \tan^{-1}v+\frac{1}{2}\int\frac{2v}{1+v^2}dv=-\log|x|+c$$

$$\Rightarrow \tan^{-1}v+\frac{1}{2}\log|1+v^2|=-\log|x|+c$$

$$\Rightarrow \tan^{-1}\frac{y}{x}+\frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right|=-\log|x|+c \qquad \text{...(iii)}$$

Given $y = 1$ when $x = 1$. Putting in equation (iii), we get

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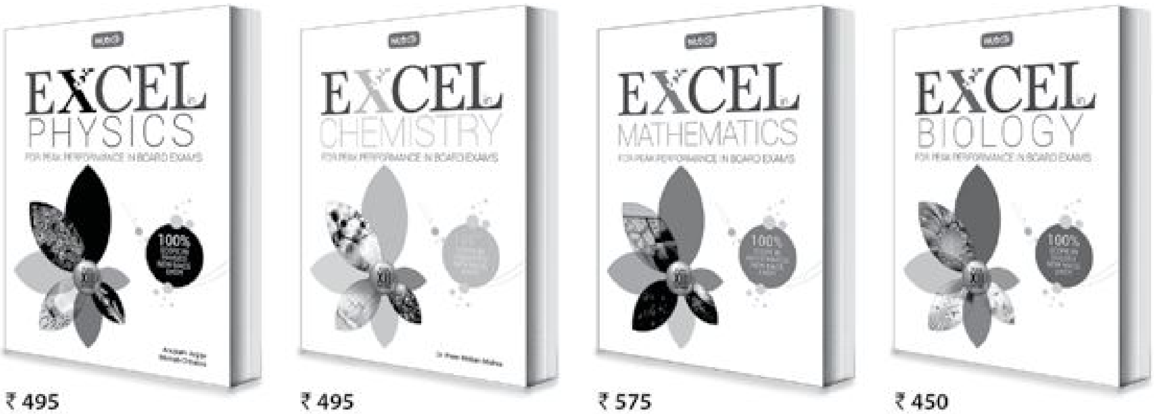

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$$\tan^{-1}1+\frac{1}{2}\log\left|1+\frac{1}{1}\right|=-\log|1|+c$$

$$c=\frac{\pi}{4}+\frac{1}{2}\log 2$$

Putting $c=\frac{\pi}{4}+\log\sqrt{2}$ in equation (iii), we get

$$\tan^{-1}\frac{y}{x}+\frac{1}{2}\log\left[1+\frac{y^2}{x^2}\right]+\log|x|=\frac{\pi}{4}+\log\sqrt{2}$$

19. The given differential equation is

$$\frac{dy}{dx}+y=\cos x-\sin x \qquad \text{...(i)}$$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P = 1$ and $Q = \cos x - \sin x$.

$$\therefore \quad \text{I.F.}=e^{\int P dx}=e^{\int 1 dx}=e^x$$

Solution of linear differential equation is

$$y\cdot(\text{I.F.})=\int Q(\text{I.F})dx+c$$

$$e^x\frac{dy}{dx}+ye^x=e^x(\cos x-\sin x)$$

$$ye^x=\int e^x(\cos x-\sin x)\,dx+c$$

$$\Rightarrow \quad ye^x=\int e^x\cos x dx-\int e^x\sin x\,dx+c$$

$$\Rightarrow \quad ye^x=e^x\cos x-\int -\sin x\,e^x\,dx-\int e^x\sin x+c$$

$$\Rightarrow \quad ye^x=e^x\cos x+\int e^x\sin x\,dx-\int e^x\sin x\,dx+c$$

$\Rightarrow \quad ye^x = e^x \cos x + c,$

Now, $y(0) = 1 \Rightarrow y = \cos 0 + c \Rightarrow c = 0$

$\therefore \quad ye^x = e^x \cos x$ is the required equation.

20. Given: $\frac{dy}{dt}\propto y\Rightarrow\frac{dy}{dt}=ky$

where k is constant of proportionality.

$$\Rightarrow \frac{dy}{y}=k\,dt$$

Integrating both sides, we get

$$\int\frac{dy}{y}=k\int dt\Rightarrow\log y=kt+c \qquad \text{...(i)}$$

Initially, when $t = 0$, $y=\frac{1}{100}$

$\therefore \quad \log\frac{1}{100} = 0 + c \Rightarrow c = \log(100)^{-1} = -\log 100$

$\therefore \quad$ From (i), we have,

$\log y = kt - \log 100 \qquad$ ...(ii)

Now it is given when $t = 10$ hours, $y=\frac{1}{300}$

$\therefore \quad \log\frac{1}{300} = 10\,k - \log 100$

$\Rightarrow \quad -\log 300 = 10\,k - \log 100$

$\Rightarrow \quad 10\,k = \log 100 - \log 300$

$$\Rightarrow \quad 10k=\log\frac{100}{300}=\log\frac{1}{3}=-\log 3 \;\therefore\; k=-\frac{\log 3}{10}$$

Putting these values in (ii), we get

$$\log y=-\frac{\log 3}{10}t-\log 100\Rightarrow\log 100+\log y=-\frac{\log 3}{10}t$$

$$\Rightarrow \log 100y=-\frac{\log 3}{10}t\Rightarrow 100y=e^{-\frac{\log 3}{10}t}$$

$$\Rightarrow y=\frac{1}{100}e^{-\frac{\log 3}{10}t}\Rightarrow y=0.01e^{-\frac{\log 3}{10}t}$$

Hence, concentration at time t is given by

$y = 0.01\,e^{-[\log 3/10]t}$

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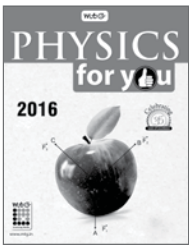

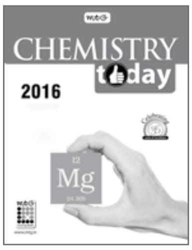

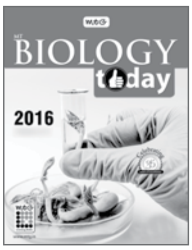

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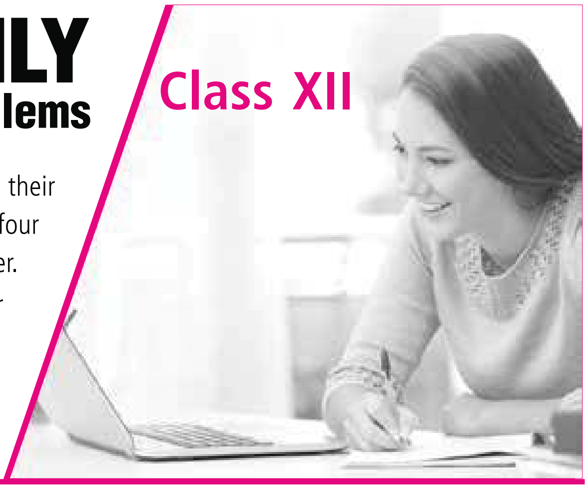

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Indefinite Integration

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $I=\int(7x+4)\sqrt{5-4x-2x^2}\,dx$, then I equals

(a) $\frac{7}{2}x\sqrt{5-4x-2x^2}+\frac{21\sqrt{2}}{4}\sin^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right)+c$

(b) $(7x+3)\sqrt{5-4x-2x^2}+\frac{21\sqrt{2}}{4}\sin^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right)+c$

(c) $\frac{1}{6}(14x^2-3x-4)\sqrt{5-4x-2x^2}+\frac{21\sqrt{2}}{4}\sin^{-1}\left(\sqrt{2}(x+1)\right)+c$

(d) None of these

2. If $I=\int(x+1)\sqrt{\frac{x+2}{x-2}}\,dx$, then I equals

(a) $\frac{1}{2}(x+6)\sqrt{x^2-4}+2\log\left|x+\sqrt{x^2-4}\right|+c$

(b) $\frac{1}{2}(x+6)\sqrt{x^2-4}+4\log\left|x+\sqrt{x^2-4}\right|+c$

(c) $\frac{1}{2}(x+6)\sqrt{x^2-4}+6\log\left|x+\sqrt{x^2-4}\right|+c$

(d) None of these

3. $\int\frac{(2x^{12}+5x^9)}{(x^5+x^3+1)^3}\,dx$ is equal to

(a) $\frac{x^2+2x}{(x^5+x^3+1)^2}+c$ (b) $\frac{x^{10}}{2(x^5+x^3+1)^2}+c$

(c) $\ln|x^5+x^3+1|+\sqrt{(2x^7+5x^4)}+c$

(d) None of these

4. If $\int\frac{\sin x}{\sin 4x}\,dx=A\log\left|\frac{1+\sin x}{1+\sin x}\right|+B\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+c$ then

(a) $A=\frac{1}{8},B=\frac{1}{4\sqrt{2}}$ (b) $A=-\frac{1}{8},B=-\frac{1}{4\sqrt{2}}$

(c) $A=-\frac{1}{8},B=\frac{1}{4\sqrt{2}}$ (d) $A=\frac{1}{8},B=-\frac{1}{4\sqrt{2}}$

5. Let $x^2+1\neq n\pi$, $n\in N$, then

$\int x\sqrt{\frac{2\sin(x^2+1)-\sin\{2(x^2+1)\}}{2\sin(x^2+1)+\sin\{2(x^2+1)\}}}\,dx$ is

(a) $\ln\left|\frac{1}{2}\sec(x^2+1)\right|+c$ (b) $\ln\left|\sec\left\{\frac{1}{2}(x^2+1)\right\}\right|+c$

(c) $\frac{1}{2}\ln|\sec(x^2+1)|+c$ (d) $\ln|\sec(x^2+1)|+c$

6. If $\int\frac{x^3}{4+x^{16}}\,dx=\frac{A}{4096}\tan^{-1}\frac{z}{\sqrt{2}}-\frac{1}{64}\log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right|+c,$

where $u=y+1/y$ and $z=y-1/y$, $y=x^4/\sqrt{2}$ then A is

(a) 128 (b) 32 (c) 64 (d) 512

One or More Than One Option(s) Correct Type

7. A primitive of $\sin 6x$ is

(a) $\frac{1}{3}(\sin^6 x-\sin^3 x)+c$

(b) $-\frac{1}{3}\cos^2 3x+c$ (c) $\frac{1}{3}\sin^2 3x+c$

(d) $\frac{1}{3}\sin\left(3x+\frac{\pi}{7}\right)\sin\left(3x-\frac{\pi}{7}\right)+c$

8. $\int\frac{x^8+4}{x^4-2x^2+2}\,dx=$

(a) $\frac{x^5}{5}+\frac{2x^3}{3}+2x+c$ (b) $\frac{x^5}{5}+\frac{x^3}{3}+2x^2+2x+c$

(c) $3x^5+10x^3+45x+c$ (d) $4x^3+5x^2+2x+c$

9. If $\int \tan^4 x\,dx = K\tan^3 x + L\tan x + f(x)$, then
(a) $K = 1/3$
(b) $L = -1$
(c) $f(x) = x + C$
(d) $K = 2/3$

10. If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log(f(x)) + c$, then $\int f(x)dx$ equals
(a) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$
(b) $ab\tan^{-1}\left(\frac{a\tan x}{b}\right)$
(c) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$
(d) $\frac{1}{ab}\tan^{-1}\left(\tan\left(\frac{bx}{a}\right)\right)$

11. If the primitive of $\sin(\ln x)$ is $f(x)\{\sin g(x) - \cos h(x)\} + c$ (c being the constant of integration), then
(a) $\lim_{x\to 2} f(x) = 1$
(b) $\lim_{x\to 1}\frac{g(x)}{h(x)} = 1$
(c) $g(e^3) = 3$
(d) $h(e^5) = 5$

12. $\int \frac{dx}{e^x\sqrt{2e^x - 1}} =$
(a) $2\sec^{-1}\sqrt{2e^x} + c$
(b) $-4\tan^{-1}\frac{1}{\sqrt{2e^x+1}} + c$
(c) $2\sec^{-1}(\sqrt{2}e^x) + c$
(d) $2\tan^{-1}\sqrt{2e^x - 1} + c$

13. If $\int \frac{x^4+1}{x^6+1}dx = \tan^{-1} f(x) - \frac{2}{3}\tan^{-1} g(x) + c$, then
(a) both $f(x)$ and $g(x)$ are odd functions
(b) $f(x)$ is monotonic function
(c) $f(x) = g(x)$ has no real roots
(d) $\int \frac{f(x)}{g(x)}dx = -\frac{1}{x} + \frac{3}{x^3} + c$

Comprehension Type

Repeated application of integration by parts gives us the reduction formula if the integrand is dependent of n, $n \in N$.

14. If $I_{m-2,\,n+2} = \int \sin^{m-2} x\,\cos^{n+2} x\,dx$ and
$$I_{m,n} = -\frac{\sin^{m-1} x\cos^{n+1} x}{(n+1)} + f(m,n)I_{m-2,\,n+2}$$ then $f(2, 3)$ is equal to
(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$
(d) $\frac{1}{5}$

15. If $I_{m,\,p} = \int x^m.(a+bx^n)^p dx$ and $I_{m,\,p} = \frac{x^{m+1}(a+bx^n)^p}{(m+1)}$ $-f(m, n, p, b)\,I_{m+n,\,p-1}$, then the value of $f(1, 2, 3, 4)$ is equal to
(a) 8
(b) 10
(c) 12
(d) None of these

Matrix Match Type

16. Match the following :

	Column I		Column II
P.	$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$	1.	$\frac{e^x}{x+2} + c$
Q.	$\int \frac{e^x(x+1)}{(x+2)^2}dx$	2.	$e^x\sqrt{\frac{1+x}{1-x}} + c$
R.	$\int \frac{x+1}{x(1+xe^x)^2}dx$	3.	$(x-2)u - \log\frac{u-1}{u+1} + c$, $u = \sqrt{e^x+1}$
		4.	$\log\left\lvert\frac{xe^x}{1+xe^x}\right\rvert + \frac{1}{1+xe^x} + c$

	P	Q	R
(a)	2	3	4
(b)	1	2	3
(c)	2	1	4
(d)	2	1	3

Integer Answer Type

17. Let $f(x) = \int x^{\sin x}(1 + x\cos x.\ln x + \sin x)\,dx$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$. Then the value of $|\cos f(\pi)|$ is

18. If $\int \frac{x dx}{\sqrt{(7x-10-x^2)^3}} = \frac{\lambda\cdot(7x-20)}{9\sqrt{7x-10-x^2}} + c$, then the value of λ is

19. If $\int x^2 \cdot e^{-2x}\,dx = e^{-2x}(ax^2 + bx + c) + d$, then the value of $|a/bc|$ is

20. If $\int\left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right]\ln x\,dx = A\left(\frac{x}{e}\right)^x + B\left(\frac{e}{x}\right)^x + C$, then the value of $A + B$ is

◈◈

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Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 178

JEE MAIN

1. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x has real roots. Then p can take any value in the interval
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$
(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

2. The bisectors of ΔABC meets its circumcircle at the points D, E, F respectively, then $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC} =$
(a) $\frac{R}{2r}$ (b) $\frac{r}{2R}$ (c) $\frac{2r}{R}$ (d) $\frac{R}{r}$

3. $\sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + \text{........}\infty =$
(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/3$

4. The curve described parametrically by $x = t^2 + t + 1$, and $y = t^2 - t + 1$ represents
(a) a pair of straight lines
(b) an ellipse
(c) a parabola
(d) a hyperbola

5. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is
(a) 216 (b) 192 (c) 58 (d) 46

JEE ADVANCED

6. Let $f : [0, 2] \to R$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$.
Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$ then $F(2)$ equals
(a) $e^2 - 1$ (b) $e^4 - 1$ (c) $e - 1$ (d) e^4

COMPREHENSION

The graph of a certain function 'f' contains the point (0, 2) and has the property that for each number t the tangent line to $y = f(x)$ at $(t, f(t))$ intersects the x-axis at $t + 2$.

7. The value of $\lim_{x\to\infty} f(x)$ is
(a) 1 (b) -1 (c) 0 (d) ∞

8. Equation of normal at (0, 2) to the curve is
(a) $x - 2y + 4 = 0$ (b) $x + 3y - 6 = 0$
(c) $x - y + 2 = 0$ (d) None of these

INTEGER TYPE

9. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from normal at any point 'P' of the curve is equal to the distance of P from the x-axis is a circle with radius

MATRIX MATCH

10. Match the following ([.] denotes the greatest integer function)

	List-I		List-II
P.	$\lim_{x\to 0}\frac{(\cos x - 1)(\cos x - e^x)}{x^n} =$	1.	does not exist
Q.	$\lim_{x\to 0} = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x} =$	2.	1/2
R.	$4\lim_{x\to\frac{3}{2}}(x - [x]) =$	3.	1/6
S.	$4\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$	4.	3

	P	Q	R	S
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	3	1	2
(d)	1	4	2	3

See Solution Set of Maths Musing 177 on page no 84

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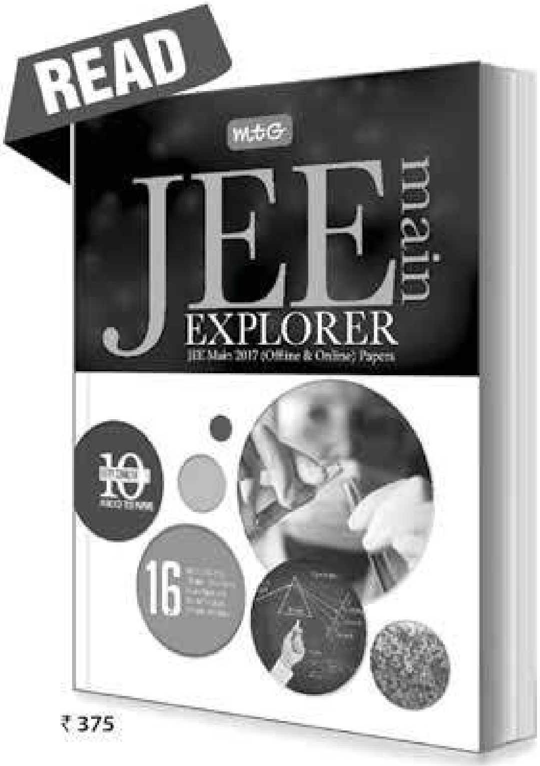
JEE Main



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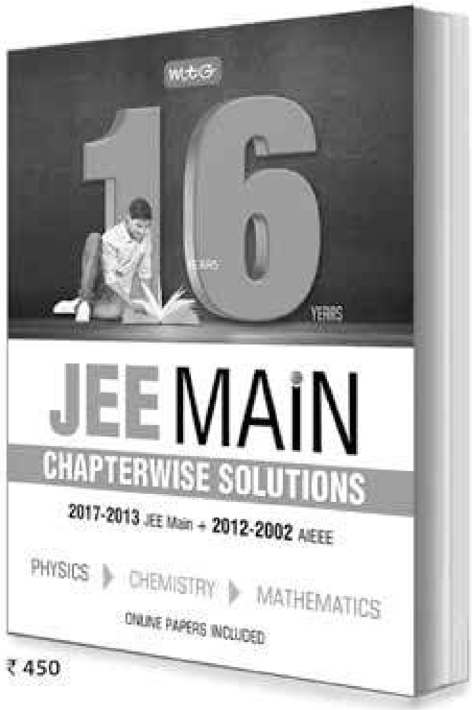

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TARGET JEE

Trigonometric Identities

Trigonometric Identities

1. If $A + B + C = \pi$, then

- $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$
- $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$
- $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$
- $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$
- $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$
- $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

2. If $A + B + C = \pi/2$, then

- $\sin^2 A + \sin^2 B + \sin^2 C = 1 - 2\sin A \sin B \sin C$
- $\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2\sin A \sin B \sin C$

Values of Trigonometric Ratios of Some Angles

1. $\sin 15° = \sin(45° - 30°) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$
2. $\cos 15° = \cos(45° - 30°) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$
3. $\tan 15° = \tan(45° - 30°) = 2-\sqrt{3} = \cot 75°$
4. $\cot 15° = \cot(45° - 30°) = 2+\sqrt{3} = \tan 75°$
5. $\tan 22\frac{1}{2}° = \tan\left(\frac{45}{2}\right)° = \sqrt{2}-1 = \cot 67\frac{1}{2}°$
6. $\cot 22\frac{1}{2}° = \cot\left(\frac{45}{2}\right)° = \sqrt{2}+1 = \tan 67\frac{1}{2}°$
7. $\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$
8. $\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$
9. $\tan 18° = \frac{\sqrt{25-10\sqrt{5}}}{5}$
10. $\cos 36° = \sin 54° = \frac{\sqrt{5}+1}{4}$
11. $\sin 36° = \cos 54° = \frac{\sqrt{10-2\sqrt{5}}}{4}$
12. $\tan 36° = \frac{\sin 36°}{\cos 36°} = \sqrt{5-2\sqrt{5}}$
13. $\sin 72° \cos 54° = \sin 36° \cos 18° = \frac{\sqrt{5}}{4}$
14. $\cos 36° \cos 72° = \sin 54° \sin 18° = \frac{1}{4}$
15. $\cos 36° - \cos 72° = \sin 54° - \sin 18° = \frac{1}{2}$

Important Series

1. $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$
2. $\tan 2x \tan(x + \alpha) \tan(x - \alpha) = \tan 2x - \tan(x + \alpha) - \tan(x - \alpha)$
3. $4\sin x \sin(60° - x) \sin(60° + x) = \sin 3x$
4. $4\cos x \cos(60° - x) \cos(60° + x) = \cos 3x$
5. $\tan x \tan(60° - x) \tan(60° + x) = \tan 3x$
6. $\cot x \cot(60° + x) \cot(120° + x) = -\frac{1}{\tan 3x}$
7. $\cos\alpha \cos 2\alpha \cos 2^2\alpha \ldots \cos 2^{n-1}\alpha$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha} & \text{if} \quad \alpha \neq n\pi \\ 1 & \text{if} \quad \alpha = 2k\pi \\ -1 & \text{if} \quad \alpha = (2k+1)\pi \end{cases}$$

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8. $\cos\alpha \cos 2\alpha \cos 2^2\alpha \ldots\ldots \cos 2^{n-1}\alpha$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}, & \text{if} \quad \alpha \neq n\pi \\ \dfrac{1}{2^n}, & \text{if} \quad \alpha = \dfrac{\pi}{2^n+1},\ n \in I \\ \dfrac{-1}{2^n}, & \text{if} \quad \alpha = \dfrac{\pi}{2^n-1},\ n \in I \end{cases}$$

Summation of Series

1. $$\sum_{r=0}^{n-1} \sin(\alpha + r\beta) = \sin\alpha + (\sin\alpha+\beta) + \sin(\alpha+2\beta) + \ldots + \sin(\alpha + (n-1)\beta)$$

$$= \frac{\sin\left(\alpha + \left(\frac{n-1}{2}\right)\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin(\beta/2)} \qquad \ldots (*)$$

If $\beta = \alpha$ then (*) reduces to

$\sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots + \sin(n\alpha)$

$$= \frac{\sin\left(\frac{n+1}{2}\right)\alpha \sin\left(\frac{n\alpha}{2}\right)}{\sin(\alpha/2)}$$

2. $$\sum_{r=0}^{n-1} \cos(\alpha + r\beta) = \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots + \cos(\alpha + (n-1)\beta)$$

$$= \frac{\cos\left(\alpha + \frac{(n-1)\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin(\beta/2)} \qquad \ldots (**)$$

If $\beta = \alpha$ then (**) reduces

$\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots + \cos(n\alpha)$

$$= \frac{\cos\left(\frac{n+1}{2}\right)\alpha \cdot \sin\left(\frac{n\alpha}{2}\right)}{\sin(\alpha/2)}$$

Properties of Inverse Trigonometric Functions

1.
- $\sin^{-1}(-x) = -\sin^{-1}(x)\ \forall\ x \in [-1, 1]$
- $\tan^{-1}(-x) = -\tan^{-1}(x)\ \forall\ x \in R$
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)\ \forall\ x \in R - (-1, 1)$
- $\cos^{-1}(-x) = \pi - \cos^{-1}(x)\ \forall\ x \in [-1, 1]$
- $\cot^{-1}(-x) = \pi - \cot^{-1}(x)\ \forall\ x \in (-\infty, \infty)$
- $\sec^{-1}(-x) = \pi - \sec^{-1}(x)\ \forall\ x \in R - (-1, 1)$

2.
- $\sin^{-1}\left(\dfrac{1}{x}\right) = \operatorname{cosec}^{-1}x\ \forall\ x \in R - (-1, 1)$
- $\cos^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1}x\ \forall\ x \in R - (-1, 1)$
- $\tan^{-1}\left(\dfrac{1}{x}\right) = \begin{cases} \cot^{-1}x, & x > 0 \\ -\pi + \cot^{-1}x, & x < 0 \end{cases}$

3.
- $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}\ \forall\ x \in [-1, 1]$
- $\sec^{-1}x + \operatorname{cosec}^{-1}x = \dfrac{\pi}{2}\ \forall\ x \in R - (-1, 1)$
- $\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}\ \forall\ x \in (-\infty, \infty)$

4. $\tan^{-1}x + \tan^{-1}y$

$$= \begin{cases} \tan^{-1}\dfrac{x+y}{1-xy} & \text{if} \quad xy < 1\ x > 0,\ y > 0 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{if} \quad x > 0,\ y > 0,\ xy > 1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{if} \quad x < 0,\ y < 0,\ xy > 1 \end{cases}$$

5. $\tan^{-1}x - \tan^{-1}y$

$$= \begin{cases} \tan^{-1}\dfrac{x-y}{1+xy} & \text{if} \quad xy > -1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{if} \quad x < 0,\ y > 0,\ xy < -1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{if} \quad x > 0,\ y < 0,\ xy < -1 \end{cases}$$

6. $\sin^{-1}x \pm \sin^{-1}y$

$$= \begin{cases} \sin^{-1}\left[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right] \forall\ x \geq 0,\ y \geq 0,\ x^2+y^2 \leq 1 \\ \pi - \sin^{-1} x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\ \forall\ x \geq 0,\ y \geq 0,\ x^2+y^2 > 1 \end{cases}$$

7.
- $\cos^{-1}x \pm \cos^{-1}y$

$$= \begin{cases} \cos^{-1}[xy \mp \sqrt{1-x^2}\ \sqrt{1-y^2}]\ \forall\ x, y \geq 0,\ x^2+y^2 \leq 1 \\ \pi - \cos^{-1}[xy \mp \sqrt{1-x^2}\ \sqrt{1-y^2}]\ \forall\ x, y \geq 0,\ x^2+y^2 > 1 \end{cases}$$

- $\cos^{-1}x - \cos^{-1}y$

$$= \begin{cases} \cos^{-1}(xy + \sqrt{1-x^2}\ \sqrt{1-y^2})\ \forall\ x, y \geq 0,\ y > x \\ -\cos^{-1}(xy + \sqrt{1-x^2}\ \sqrt{1-y^2}]\ \forall\ x, y \geq 0,\ y < x \end{cases}$$

8. $$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\dfrac{2x}{1+x^2}\right), & \text{if } |x| \leq 1 \\ \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \text{if } |x| < 1 \\ \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), & \forall\ x > 1 \\ \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) & \forall\ x \in [0, 1] \\ -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & \text{if } x \in (-\infty, -1) \end{cases}$$

9. • $\sin^{-1}(2x\sqrt{1-x^2})$

$$=\begin{cases} 2\sin^{-1}x, & \text{if} \quad |x|\le\frac{1}{\sqrt{2}} \\ \pi-2\sin^{-1}x, & \text{if} \quad \frac{1}{\sqrt{2}}\le x\le 1 \\ -\pi-2\sin^{-1}x, & \text{if} \quad -1\le x\le -\frac{1}{\sqrt{2}} \end{cases}$$

• $\cos^{-1}(2x^2-1) = \begin{cases} 2\cos^{-1}x, & \text{if } x\in[0,1] \\ 2\pi-2\cos^{-1}x, & \text{if } x\in[-1,0] \end{cases}$

• $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases} 2\tan^{-1}x, |x|\le 1 \\ \pi-2\tan^{-1}x, x>1 \\ -\pi-2\tan^{-1}x, x<-1 \end{cases}$

• $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\begin{cases} 2\tan^{-1}x, & \forall\ x\in[0,\ \infty) \\ -2\tan^{-1}x, & \forall\ x\in(-\infty,\ 0] \end{cases}$

• $\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\begin{cases} 2\tan^{-1}x,\ \forall\ |x|\le 1 \\ \pi-2\tan^{-1}x,\ \forall\ x\in(-\infty,\ -1) \\ -\pi+2\tan^{-1}x,\ \forall\ x\in(1,\infty) \end{cases}$

10. $\cot^{-1}x=\begin{cases} \tan^{-1}\left(\frac{1}{x}\right), \text{ if } x>0 \text{ i.e. } \cot^{-1}x\in\left[0,\ \frac{\pi}{2}\right) \\ \pi+\tan^{-1}\left(\frac{1}{x}\right), \text{ if } x<0 \text{ i.e. } \cot^{-1}x\in\left(\frac{\pi}{2},\ \pi\right) \end{cases}$

11. • $\sin^{-1}(3x-4x^3)=\begin{cases} 3\sin^{-1}x,\ \forall\ |x|\le\frac{1}{2} \\ \pi-3\sin^{-1}x,\ \forall\ x\in\left(\frac{1}{2},\ 1\right] \\ -\pi-3\sin^{-1}x,\ \forall\ x\in\left[-1,\ -\frac{1}{2}\right) \end{cases}$

• $\cos^{-1}(4x^3-3x)=\begin{cases} -2\pi+3\cos^{-1}x,\ \forall\ x\in\left[-1,\ -\frac{1}{2}\right] \\ 2\pi-3\cos^{-1}x,\ \forall\ |x|\le\frac{1}{2} \\ 3\cos^{-1}x,\ \forall\ x\in\left[\frac{1}{2},\ 1\right] \end{cases}$

• $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)=\begin{cases} \pi+3\tan^{-1}x, \forall x<\frac{-1}{\sqrt{3}} \\ 3\tan^{-1}x, \forall|x|<\frac{1}{\sqrt{3}} \\ -\pi+3\tan^{-1}x, \forall x\in\left(\frac{1}{\sqrt{3}},\infty\right) \end{cases}$

12. • $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$$=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right),\ x,\ y,\ z\ge 0$$

• $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \dots + \tan^{-1}x_n$

$$=\tan^{-1}\left[\frac{S_1-S_3+S_5-S_7+\dots}{1-S_2+S_4-S_6+\dots}\right]$$

where $S_1=\sum_{i=1}^{n}x_i, (i=1,\dots,n)\in R$

$S_2=x_1x_2+x_2x_3+\dots+x_{n-1}x_n=\sum x_1x_2$

$S_3=\sum x_1\,x_2\,x_3$ and so on.

Conversion of Inverse Trigonometric Function in Terms of Others

1. • If $a\sin^{-1}x + b\cos^{-1}x = c$, then

$$a\sin^{-1}x-b\cos^{-1}x=\frac{c(a+b)-\pi ab}{a-b}$$

• If $a\tan^{-1}x + b\cot^{-1}x = c$, then

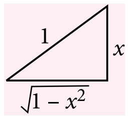
$$b\tan^{-1}x+a\cot^{-1}x=\frac{\pi(a+b)-2c}{2}$$

• If $a\sec^{-1}x + b\operatorname{cosec}^{-1}x = c$, then

$$b\sec^{-1}x-a\operatorname{cosec}^{-1}x=\frac{2c(a+b)-\pi(a^2+b^2)}{2(a-b)}$$

2. $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}$

$\tan^{-1}\frac{x}{\sqrt{1-x^2}}=\sec^{-1}\frac{1}{\sqrt{1-x^2}}$



$\operatorname{cosec}^{-1}\left(\frac{1}{x}\right)=\cot^{-1}\frac{\sqrt{1-x^2}}{x}$

Similarly for $\cos^{-1}x$ and $\tan^{-1}x$.

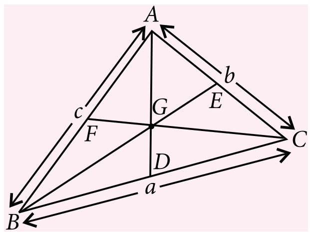
Medians and Centroid of Triangle

1. The distances of centroid from the vertices A, B, C of the triangle ABC are GA, GB and GC respectively given by

• $GA=\frac{\sqrt{2b^2+2c^2-a^2}}{3}$

• $GB=\frac{\sqrt{2a^2+2c^2-b^2}}{3}$

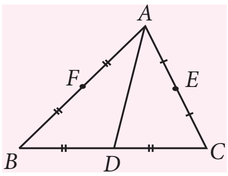
• $GC=\frac{\sqrt{2a^2+2b^2-c^2}}{3}$



2. Distances of centroid (G) from the sides BC, CA and AB respectively given by

• $G_a=\frac{2\times\text{ area of triangle}}{3a}=\frac{2\Delta}{3a}$

- $G_b = \dfrac{2\Delta}{3b}$ • $G_c = \dfrac{2\Delta}{3c}$
- $AD = \dfrac{1}{2}\sqrt{b^2 + c^2 + 2bc\cos A}$
- $BE = \dfrac{1}{2}\sqrt{c^2 + a^2 + 2ac\cos B}$
- $CF = \dfrac{1}{2}\sqrt{a^2 + b^2 + 2ab\cos C} = \dfrac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$



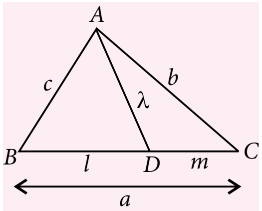
Bisectors of Angles

If AD is bisector of angle A and divide the opposite sides BC in two parts l and m then by geometry we have

$$\frac{l}{m} = \frac{AB}{AC} = \frac{c}{b}$$

$$\therefore \frac{l}{c} = \frac{m}{b} = \frac{l+m}{b+c} = \frac{a}{b+c}$$

$$\Rightarrow l = \frac{ac}{b+c} \text{ and } m = \frac{ab}{a+c}$$



Let $AD = \lambda$ then we have

$ar(\Delta ABD) + ar(\Delta ACD) = ar(\Delta ABC)$

$$\Rightarrow \frac{c}{2}\lambda\sin\frac{A}{2} + \frac{b}{2}\lambda\sin\frac{A}{2} = \frac{1}{2}bc\sin A$$

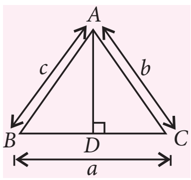
$$\Rightarrow \lambda = \text{length of } AD = \frac{bc\sin A}{(b+c)\sin\dfrac{A}{2}}$$

$$= \frac{2bc\cos(A/2)}{b+c} \quad \left(\text{using } \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}\right)$$

Properties of Triangles and Circles

1. Sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



2. Cosine formula

- $\cos B = \dfrac{c^2 + a^2 - b^2}{2ac}$
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.
- $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

3. Projection formula

- $a = c\cos A + b\cos C$
- $b = a\cos C + c\cos A$
- $c = a\cos B + b\cos A$

4. Half angle formula

- $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ • $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$
- $\sin\dfrac{B}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ac}}$ • $\cos\dfrac{B}{2} = \sqrt{\dfrac{s(s-b)}{ac}}$
- $\sin\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$ • $\cos\dfrac{C}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$

5. Area of ΔABC in different forms

If Δ denotes the area of triangle ABC

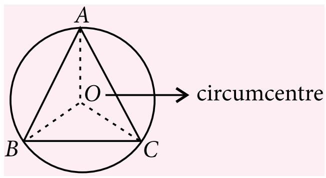
- $\Delta = \dfrac{c^2\sin A\sin B}{2\sin C} = \dfrac{b^2\sin A\sin C}{2\sin B} = \dfrac{a^2\sin B\sin C}{2\sin A}$
- $\Delta = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}ac\sin B = \dfrac{1}{2}bc\sin A$

6. Tangent rule (Napier's analogy)

- $\tan\left(\dfrac{B-C}{2}\right) = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$
- $\tan\left(\dfrac{C-A}{2}\right) = \dfrac{c-a}{c+a}\cot\dfrac{B}{2}$
- $\tan\left(\dfrac{A-B}{2}\right) = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$

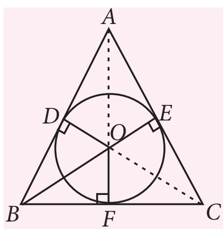
Circumcircle and Incircle

- Circumcircle of a triangle and circumradius (R)
  $OA = OB = OC = R$, where O is point of intersection of perpendicular bisector of sides of the triangle ABC



1. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$
2. $R = \dfrac{abc}{4\Delta} = \dfrac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$

- **Inscribed Circle and Inradius**
  O is point of intersection of internal bisector known as incentre (r).
  $OF = OE = OD$ = length of perpendicular from incentre O to any one of the side of the triangle.



1. $r = \dfrac{\Delta}{s}$ *i.e.* $\Delta = rs$

2. $r=(s-a)\tan\frac{A}{2}, r=(s-b)\tan\frac{B}{2},\ r=(s-c)\tan\frac{C}{2}$

$\therefore\ r=(s-a)\tan\frac{A}{2}=(s-b)\tan\frac{B}{2}=(s-c)\tan\frac{C}{2}$

$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

3. $r=\dfrac{a\sin(B/2)\sin(C/2)}{\cos(A/2)},\ r=\dfrac{b\sin(A/2)\sin(C/2)}{\cos(B/2)}$

$r=\dfrac{c\sin(A/2)\sin(B/2)}{\cos(A/2)}$

4. Let r_1, r_2, r_3 are exradius and a, b, c are sides of triangle ABC then

- $r_1=\dfrac{\Delta}{s-a}=\sqrt{\dfrac{s(s-b)(s-c)}{(s-a)}}, r_2=\dfrac{\Delta}{s-b}=\sqrt{\dfrac{s(s-a)(s-c)}{s-b}},$

  $r_3=\dfrac{\Delta}{s-c}=\sqrt{\dfrac{s(s-a)(s-b)}{s-c}}$

  Thus, $r_1=\dfrac{\Delta}{s-a}=s\tan\dfrac{A}{2}=\dfrac{a\cos(B/2)\cos(C/2)}{\cos(A/2)}$

  $=4R\cos\frac{B}{2}\cos\frac{C}{2}\sin\frac{A}{2}$

  $r_2=4R\cos\frac{A}{2}\cos\frac{C}{2}\sin\frac{B}{2}, r_3=4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

- $\dfrac{1}{r}=\dfrac{s}{\Delta}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}$
- $r_1+r_2+r_3=4R+r$
- $r\cdot r_1r_2r_3=\Delta^2$
- $\dfrac{1}{r^2}+\dfrac{1}{r_1^2}+\dfrac{1}{r_2^2}+\dfrac{1}{r_3^2}=\dfrac{a^2+b^2+c^2}{\Delta^2}$

5. If r_1, r_2, r_3 are given then we can find

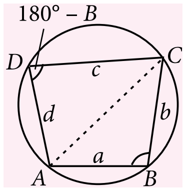
- $s=\sqrt{r_1r_2+r_2r_3+r_1r_3}$
- $\Delta=\dfrac{r_1r_2r_3}{\sqrt{\sum r_1r_2}}, r=\dfrac{r_1r_2r_3}{\sum r_1r_2}$
- $R=\dfrac{(r_1+r_2)(r_2+r_3)(r_3+r_1)}{4(r_1r_2+r_2r_3+r_3r_1)}$
- $a=\dfrac{r_1(r_2+r_3)}{\sqrt{\sum r_1r_2}},\ b=\dfrac{r_2(r_3+r_1)}{\sqrt{\sum r_1r_2}},\ c=\dfrac{r_3(r_2+r_1)}{\sqrt{\sum r_1r_2}}$
- $\sin B=\dfrac{2r_2\sqrt{\sum r_1r_2}}{(r_2+r_3)(r_2+r_1)},\ \sin C=\dfrac{2r_3\sqrt{\sum r_1r_2}}{(r_3+r_1)(r_3+r_2)}$

  and $\sin A=\dfrac{2r_1\sqrt{\sum r_1r_2}}{(r_1+r_2)(r_1+r_3)}$

Area of Cyclic Quadrilateral

Let $ABCD$ is a cyclic quadrilateral with sides $AB = a$, $BC = b$, $CD = c$, $DA = d$ then $\angle A+\angle C=\angle B+\angle D=180°$.

1. Area of cyclic quadrilateral



$=\frac{1}{2}ab\sin B+\frac{1}{2}cd\sin D$

$=\frac{1}{2}(ab+cd)\sin B$

$(\because\ \angle D=180°-B)$

$=\sqrt{(s-a)(s-b)(s-c)(s-d)}$ ...(i)

where $s=\dfrac{a+b+c+d}{2}$

2. Again, area of cyclic quadrilateral

$=\frac{1}{2}(ab+cd)\sin B=\frac{1}{2}(ab+cd)\sin D$

$\therefore\ \sin B=\dfrac{2\cdot\text{area of cyclic quadrilateral}}{ab+cd}$

3. $\cos B=\dfrac{(a^2+b^2)-(c^2+d^2)}{2(ab+cd)}$

   (Use this result, if three sides and included angle of one is known)

4. $(AC)^2 = a^2 + b^2 - 2ab\cos B$

$=a^2+b^2-2ab\left[\dfrac{(a^2+b^2)-(c^2+d^2)}{2(ab+cd)}\right]$

$(AC)^2=\dfrac{(ac+bd)(ad+bc)}{ab+cd}$ ... (ii)

and $(AC)^2 = d^2 + c^2 - 2cd\cos D = d^2 + c^2 + 2cd\cos B$

$=d^2+c^2+2cd\left[\dfrac{(a^2+b^2)-(c^2+d^2)}{2(ab+cd)}\right]$ ... (iii)

$\therefore$ On equating (ii) and (iii) one get (i) area of quadrilateral.

5. Circumradius of a cyclic quadrilateral

$R=\dfrac{\sqrt{(ac+bd)(ad+bc)(ab+cd)}}{4\Delta}$

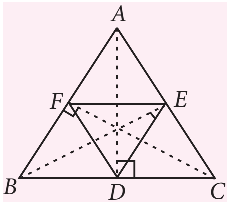
6. If A, B, C, D taken in order then in a cyclic quadrilateral $ABCD$ we have

   $\cos A + \cos B + \cos C + \cos D = 0$

Concept of Pedal Triangle

Let ABC be any triangle and AD, BE, CF are perpendicular's drawn from A, B, C to the opposite sides BC, CA, AB respectively. The point of concurrency of these perpendiculars (altitudes) of the triangle ABC is called orthocentre of the triangle.

The triangle formed by joining the feet of these perpendiculars is called pedal triangle. In the figure, ΔDEF is the pedal triangle of ΔABC.



$\therefore$ Sides of pedal triangle are

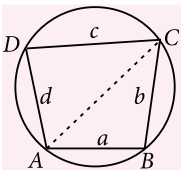
- $DE = c\cos C = 2R\sin C \cos C = R \sin 2C$
- $EF = a\cos A = 2R\sin A \cos A = R \sin 2A$
- $DF = b\cos B = 2R\sin B \cos B = R \sin 2B$

$$\left(\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R\right)$$

- The inradius of pedal triangle = $2R\cos A \cos B \cos C$
- Circumradius of pedal triangle = $R/2$
- Perimeter of pedal triangle = $a\cos A + b\cos B + c\cos C$
- Area of pedal triangle = $2\Delta \cos A \cos B \cos C = \frac{1}{2} R^2 \sin 2A \sin 2B \sin^2 C$

Ptolemy's Theorem

In a cyclic quadrilateral, the product of the diagonals is equal to the sum of the products of the lengths of opposite sides.



$$\left.\begin{aligned} &\textit{i.e.}\ \ AC \cdot BD = ac + bd \\ &\text{or}\ \ AC \cdot BD = AB \cdot CD + BC \cdot BD \end{aligned}\right\}\ ^{*})$$

PROBLEMS

Single Correct Answer Type

1. If $\cos^{-1}\sqrt{x^2+2x+1}+\operatorname{cosec}^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$, $x \neq 0$, then value of $\sec^{-1}\frac{x}{2}-\sin^{-1}\left(\frac{x}{2}\right)$ is equal to

(a) $-\frac{3\pi}{2}$ (b) $-\frac{\pi}{2}$ (c) $3\pi/2$ (d) $\pi/2$

2. If $(\cot^{-1}x)^2 - 9\cot^{-1}x + 20 > 0$, then x lies is

(a) $(\cot 5, \cot 4)$
(b) $(-\infty, \cot 5) \cup (\cot 4, \infty)$
(c) $(\cot 4, \infty)$ (d) None of these

3. If the sum of the roots of the equation $2\sin^2 2x + 7\cos 2x - 7 = 0$ over the interval $[0, 2017]$ is $m\pi$ then the value of $[\sqrt{m}]$ (where $[\cdot]$ represents greatest integer function) is

(a) 453 (b) 454 (c) 205761 (d) 642

4. The number of solutions of $\sum_{r=1}^{2017} \cos r\, x = 2017$ in the interval $[0, 2\pi]$ is

(a) 2018 (b) 2017
(c) 2015 (d) None of these

5. The general solution of the equation $\sum_{\lambda=1}^{n} \cos(\lambda^2 x)\sin(\lambda x)=\frac{1}{2}$ is

(a) $\frac{(4p+1)}{n(n+1)}\frac{\pi}{2}, p \in I$ (b) $\frac{4p-1}{n(n+1)}\frac{\pi}{2}, p \in I$

(c) $\frac{4p+1}{n(n+1)}\pi$ (d) None of these

6. If $\log_{\cos x}\sin x + \log_{\sin x}\cos x = 2$, then value of x equals

(a) $(2p+1)\frac{\pi}{4}$ (b) $\frac{p\pi}{4}$

(c) $(4p+1)\frac{\pi}{4}$ (d) $(p+1)\frac{\pi}{4}$

7. If ex-radii r_1, r_2, r_3 of a triangle are in H.P then sides a, b, c are in

(a) A.G.P. (b) A.P.
(c) G.P. (d) None of these

8. A triangle is inscribed in a circle. The vertices of the triangle divided the circumference of the circle into three area of length 6, 8, 10 units then the area of triangle is equal to (in sq. units)

(a) $\frac{64\sqrt{3}(\sqrt{3}+1)}{\pi^2}$ (b) $\frac{72\sqrt{3}}{\pi^2}(\sqrt{3}+1)$

(c) $\frac{36\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ (d) $\frac{36\sqrt{3}}{\pi^2}(\sqrt{3}+1)$

9. The number of real roots of the equation $\sec\theta + \operatorname{cosec}\theta = \sqrt{15}$ lying between 0 and 2π is/are

(a) 0 (b) 2 (c) 4 (d) 8

10. The number of roots of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(a) 3 (b) 2 (c) 1 (d) 0

More than One Correct Answer Type

11. If $f_n(\alpha) = \tan\frac{\alpha}{2}$ $(1 + \sec\alpha)(1 + \sec 2\alpha)$ $(1 + \sec 4\alpha)$ .....$(1 + \sec 2^n\alpha)$, $n \in N$ then

(a) $f_2\left(\frac{\pi}{16}\right)=f_5\left(\frac{\pi}{128}\right)$ (b) $f_3\left(\frac{\pi}{32}\right)=f_4\left(\frac{\pi}{64}\right)$

(c) $f_2\left(\frac{\pi}{16}\right)=f_4\left(\frac{\pi}{64}\right)$

(d) $f_2\left(\frac{\pi}{16}\right)=f_3\left(\frac{\pi}{32}\right)=f_4\left(\frac{\pi}{64}\right)=f_5\left(\frac{\pi}{128}\right)=1$

12. If $y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$ then

(a) y has least value $\frac{-\pi^3}{8}$

(b) y has greatest value $\frac{7\pi^3}{8}$

(c) y has greatest value $\frac{5\pi^3}{16}$

(d) y has least value $\frac{\pi^3}{32}$

13. If $x = a\sin^{1008}\alpha\cos^{1009}\alpha$, $y = a\sin^{1009}\alpha\cos^{1008}\alpha$ and $\frac{(x^2+y^2)^l}{(xy)^m}, (l, m \in N)$ is independent of α, then

(a) $l = 2017$ (b) $m = 2017$

(c) $l = 2016$ (d) $m = 2016$

14. If $(1 + ax)^n = 1 + 16x + 112x^2 + ...$ and m = number of positive integral solutions of the equation $\tan^{-1}x + \cot^{-1}y = \tan^{-1}5$, then which of the following is true?

(a) $m = 2$ (b) $n = 8$

(c) $a = 2$ (d) $\frac{n}{ma} = 2$

15. Let $f(\alpha) = e^{\cos^{-1}\left(\sin\left(\alpha+\frac{\pi}{3}\right)\right)}$ then

(a) $f\left(\frac{8\pi}{9}\right) = e^{\frac{13\pi}{18}}$ (b) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{\pi}{12}}$

(c) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{11\pi}{12}}$ (d) $f\left(\frac{8\pi}{9}\right) = e^{\frac{5\pi}{18}}$

16. If $\tan 3A = \lambda \tan A$, $A \neq 1$ then

(a) $\lambda < \frac{1}{3}$ (b) $\frac{\sin 3A}{\sin A} = \frac{2\lambda}{\lambda - 1}$

(c) $\lambda > 3$ (d) $\frac{\cos A}{\cos 3A} = \frac{\lambda - 1}{2}$

17. The solution of the system of equations $x + y = \frac{\pi}{2}$ and $\sin x + \sin y = \sqrt{2}$ satisfies by

(a) $x = 2k\pi + \frac{\pi}{4}, k \in I$

(b) $x = 2k\pi - \frac{\pi}{2}, k \in I$

(c) $y = \frac{\pi}{4} - 2k\pi, k \in I$

(d) $x = 2k\pi + \frac{\pi}{3}, k \in I$

Comprehension Type

Paragraph for Q.No. 18 to 20

Let $\cos A \cos 2A \cos 4A \cos 8A$ ....$\cos 2^{n-1} A$

$$\begin{cases} \dfrac{\sin 2^n A}{2^n \sin A}, \text{if } A \neq n\pi \\ \dfrac{1}{2^n}, \text{if } A = \dfrac{\pi}{2^n+1} \\ -\dfrac{1}{2^n}, \text{if } A = \dfrac{\pi}{2^n-1} \forall n \in N \end{cases}$$

18. The value of $\prod_{r=1}^{4} \cos 2^r A \left(\text{if } A = \frac{\pi}{15}\right)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{16}$

19. The value of $\prod_{r=1}^{7} \sin(2r-1)A \left(\text{if } A = \frac{\pi}{14}\right)$ is

(a) $-\frac{1}{64}$ (b) $\frac{1}{64}$ (c) $\frac{1}{32}$ (d) $-\frac{1}{32}$

20. If $A = \frac{\pi}{7}$ then value of $\prod_{r=1}^{3} \cos 2rA$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Paragraph for Q.No. 21 to 23

If $\sum_{r=1}^{n} \tan^{-1}\left(\frac{x_r - x_{r-1}}{1 + x_{r-1}x_r}\right) =$

$$\sum \tan^{-1} x_r - \tan^{-1} x_{r-1} = \tan^{-1} x_n - \tan^{-1} x_0 \quad \forall n \in N$$

21. The sum of infinite terms of the series $\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + ...\infty$ is equal to

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}2$ (c) $\cot^{-1}2$ (d) $\frac{\pi}{2}$

22. The value of the infinite terms of the series

$$\tan^{-1}\left(\frac{2}{2+1^2+1^4}\right) + \tan^{-1}\left(\frac{4}{2+2^2+2^4}\right) + \tan^{-1}\left(\frac{6}{2+3^2+3^4}\right) + ...\text{is}$$

(a) $\frac{3\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

23. The value of the infinite forms of the series

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{4}{33}\right)+...+\tan^{-1}\left(\frac{8}{129}\right)+\tan^{-1}\left(\frac{16}{513}\right)+\tan^{-1}\left(\frac{32}{2049}\right)...\infty$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 2π (d) π

Paragraph for Q.No. 24 to 26

Let AD, BE, CF are perpendicular from angular points of a triangle ABC on the opposite sides let Δ, r, R respectively represents the area, in radius and circumradius of the triangle.

24. If perimeters of ΔABC and ΔDEF are P_1 and P_2 respectively then the value of $\frac{P_1}{P_2}$ equals

(a) $\frac{\Delta}{rR}$ (b) $\frac{r}{R}$ (c) $\frac{rR}{\Delta}$ (d) $\frac{R}{r}$

25. If area of Δ's AEF, BFD and CDE are Δ_1, Δ_2 and Δ_3 respectively then $\Delta_1 + \Delta_2 + \Delta_3$ equals

(a) $2\Delta(1 + \cos A \cos B \cos C)$
(b) $2\Delta(1 + \sin A \sin B \sin C)$
(c) $\Delta(1 - 2\cos A \cos B \cos C)$
(d) $\Delta(1 - 2 \sin A \sin B \sin C)$

26. If area of ΔDEF is Δ' then value of $\frac{\Delta'}{\Delta}$ equals

(a) $4\cos A \cos B \cos C$ (b) $2\cos A \cos B \cos C$
(c) $4\sin A \sin B \sin C$ (d) $2\sin A \sin B \sin C$

Matrix-Match Type

27. Match the trigonometric ratio with the equations whose one of the roots is given

Column-I		Column-II	
A.	$\cos 6°$	1.	$8x^3 - 6x - 1 = 0$
B.	$2\cos 10°$	2.	$x^3 - 3x^2 - 3x + 1 = 0$
C.	$\tan 15°$	3.	$x^6 - 6x^4 + 9x^2 - 3 = 0$
D.	$\cos 20°$	4.	$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$

28. Match the following.

Column-I		Column-II	
A.	$\cos^{-1}\frac{3}{5}+2\tan^{-1}\frac{1}{3}$	1.	π
B.	$\cos^{-1}\frac{5}{13}+\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{63}{16}$	2.	$\pi/6$
C.	$\sin^{-1}\frac{1}{5\sqrt{2}}+2\tan^{-1}\frac{1}{3}$	3.	$\pi/2$
D.	If $A=\tan^{-1}\frac{x\sqrt{3}}{2\lambda-x}$ and $B=\tan^{-1}\frac{2x-\lambda}{\lambda\sqrt{3}}$ then $A - B$ is	4.	$\pi/4$

Integer Type

29. If in a ΔABC, $BC = 5$, $CA = 4$, $AB = 3$ and D, E are the points on the side BC such that $BD = DE = EC$ then the value of $8\tan(\angle CAE)$ is

30. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and angle between them is 60°. If area of the quadrilateral is $4\sqrt{3}$ then find the length of larger of the rest two sides.

31. If $\alpha + \beta + \gamma = \pi$ and

$$\tan\left(\frac{\alpha+\beta-\gamma}{4}\right)\tan\left(\frac{\gamma+\alpha-\beta}{4}\right)\tan\left(\frac{\gamma+\beta-\alpha}{4}\right)=1 \text{ then}$$

the value of $1 + \cos\alpha + \cos\beta + \cos\gamma$ is $(m - 1)$ thus the value of m equals

SOLUTIONS

1. **(c) :** $\because\ \cos^{-1}\sqrt{x^2+2x+1}+\operatorname{cosec}^{-1}\sqrt{x^2+2x+1}=\frac{\pi}{2}$

$$\Rightarrow \cos^{-1}\sqrt{x^2+2x+1}+\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+1}}\right)=\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\sqrt{x^2+2x+1}-\cos^{-1}\left(\frac{1}{\sqrt{x^2+2x+1}}\right)=0$$

$$\Rightarrow \cos^{-1}|x+1|-\cos^{-1}\frac{1}{|x+1|}=0\ (x\neq-1)$$

$$\Rightarrow |x+1|=\frac{1}{|x+1|} \Rightarrow |x+1|^2=1$$

or, $x+1=\pm1 \Rightarrow x=-2,\ 0$

but $x \neq 0 \Rightarrow x = -2$

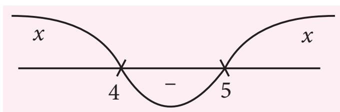
Then, $\sec^{-1}\left(\frac{x}{2}\right)-\sin^{-1}\left(\frac{x}{2}\right)$

$$=\sec^{-1}(-1)-\sin^{-1}(-1)=\pi+\frac{\pi}{2}=\frac{3\pi}{2}$$

2. **(b) :** $\because\ (\cot^{-1}x)^2 - 9\cot^{-1}x + 20 > 0$

$$\Rightarrow (\cot^{-1}x-5)(\cot^{-1}x-4)>0$$

$$\Rightarrow \cot^{-1}x<4 \text{ and } \cot^{-1}x>5$$



As $\cot^{-1}x$ is decreasing function then
$x \in (-\infty, \cot 5) \cup (\cot 4, \infty)$

3. (a) : ∵ $2\sin^2 2x + 7\cos 2x - 7 = 0$

$\Rightarrow \ 2(1-\cos^2 2x)+7\cos 2x-7=0$

$\Rightarrow \ 2\cos^2 2x-7\cos 2x+5=0$

$\Rightarrow \ 2t^2-7t+5=0$, where $t=\cos 2x$

$\Rightarrow \ (t-1)(2t-5)=0 \Rightarrow t=1,\ t=\frac{5}{2}$ (rejected)

$\therefore \ t=1 \Rightarrow \cos 2x = 1 = \cos 0°$

$\Rightarrow \ 2x=2n\pi \Rightarrow x=n\pi,\ n\in I$

∴ The roots over the interval [0, 2017] are
$0, 2\pi, 3\pi, \ldots, 641\pi$, (∵ $642\pi > 2017$)

$\Rightarrow$ Sum of roots $= \pi + 2\pi + 3\pi + \ldots + 641\pi$
$= 205761\pi = m\pi$ (given)

$\therefore \ m = 205761 \Rightarrow [\sqrt{m}] = 453$

4. (d) : ∵ $\sum_{r=1}^{2017} \cos rx = 2017$

$\Rightarrow \cos x+\cos 2x+\cos 3x+\cos 4x+\cos 5x+\ldots\ldots+\cos 2017x$ $= 2017$, which is possible only if $\cos x = \cos 2x = \ldots$ $= \cos 2017\alpha = 1$ and is satisfied for $x = 0$ and $x = 2\pi$.

∴ Number of solutions is 2.

5. (a) : ∵ Given, $\sum_{\lambda=1}^{n} 2\cos(\lambda^2 x)\cdot\sin(\lambda x)=1$

$\Rightarrow \ \sum_{\lambda=1}^{n}[\sin\{\lambda(\lambda+1)\}x-\sin\{\lambda(\lambda-1)\}x]=1$

Putting $\lambda = 1, 2, \ldots, n-1, n$

$\Rightarrow$ $(\sin 2x - \sin 0) + (\sin 6x - \sin 2x) + (\sin 12x - \sin 6x)$ $\ldots + \{\sin((n-1)n)x - \sin(n-2)(n-1)x\} + \{\sin(n(n+1))x - \sin(n(n-1))x\}$

$\Rightarrow \ \sin[n(n+1)x]-\sin 0=1 \Rightarrow \sin n(n+1)x=1$

$\Rightarrow \ n(n+1)x=2p\pi+\frac{\pi}{2},\ p\in I \Rightarrow x=\frac{(4p+1)}{n(n+1)}\cdot\frac{\pi}{2}, p\in I$

6. (c) : $\log_{\cos x}(\sin x) + \log_{\sin x}(\cos x) = 2$

$\Rightarrow \ \frac{\log\sin x}{\log\cos x}+\frac{\log\cos x}{\log\sin x}=2 \left(\text{Using } \log_a b=\frac{\log b}{\log a}\right)$

$\Rightarrow \ m+\frac{1}{m}=2 \ \left(\text{where } m=\frac{\log(\sin x)}{\log(\cos x)}\right)$

$\Rightarrow \ m^2-2m+1=0 \ \therefore m=1$

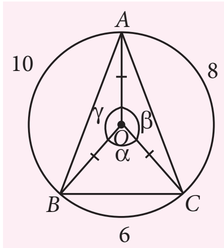
$\therefore \ \frac{\log\sin x}{\log\cos x}=1 \Rightarrow \log(\sin x) - \log(\cos x) = 0$

$\Rightarrow \log(\tan x)=0 \ \therefore \tan x=e^0=1=\tan\frac{\pi}{4}$

$\Rightarrow \ x=p\pi+\frac{\pi}{4}=(4p+1)\frac{\pi}{4}$

7. (b)

8. (d) : From problem, we have assumed that



$\overset{\frown}{AB}=10, \overset{\frown}{CA}=8, \overset{\frown}{BC}=6$

Let 'O' be the centre of circumcircle of the ΔABC and $OA = OB = OC = r$ be circumradii of the circle.

$\therefore \ r\alpha = 6,\ r\beta = 8,\ r\gamma = 10$ $\left(\because \text{Angle}=\frac{\text{arc}}{\text{radius}}\right)$

$\therefore \ r\alpha + r\beta + r\gamma = 6 + 8 + 10$

$\Rightarrow \ r(\alpha+\beta+\gamma)=24 \Rightarrow r(2\pi)=24 \Rightarrow r=\frac{12}{\pi}$

∴ Area of triangle $ABC = ar(\Delta OBC) + ar(\Delta OCA) + ar(\Delta OAB)$

$=\frac{1}{2}r^2\sin\alpha+\frac{1}{2}r^2\sin\beta+\frac{1}{2}r^2\sin\gamma$

$=\frac{1}{2}r^2(\sin\alpha+\sin\beta+\sin\gamma)$

$=\frac{1}{2}r^2\left(\sin\left(\frac{6}{r}\right)+\sin\left(\frac{8}{r}\right)+\sin\left(\frac{10}{r}\right)\right)$

$=\frac{1}{2}\left(\frac{12}{\pi}\right)^2\left[\sin\frac{6}{12}\pi+\sin\left(\frac{8}{12}\pi\right)+\sin\left(\frac{10}{12}\pi\right)\right]$

$=\frac{72}{\pi^2}\left[\sin\frac{\pi}{2}+\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}\right]$

$=\frac{72}{\pi^2}[\sin 90°+\sin 120°+\sin 150°]$

$=\frac{72}{\pi^2}\left(1+\frac{\sqrt{3}}{2}+\frac{1}{2}\right)=\frac{36\sqrt{3}}{\pi^2}(\sqrt{3}+1)$

9. (c) : ∵ $\sec\theta+\operatorname{cosec}\theta=\sqrt{15}$

$\Rightarrow \ \sin\theta+\cos\theta=\sqrt{15}\sin\theta\cos\theta$

$\Rightarrow \ 1+\sin 2\theta=\frac{15}{4}\sin^2 2\theta$ (after squaring)

$\Rightarrow 15\sin^2 2\theta-4\sin 2\theta-4=0$

$\Rightarrow (3\sin 2\theta-2)(5\sin 2\theta+2)=0$

$\Rightarrow \sin 2\theta=\frac{2}{3}$ and $\sin 2\theta=-\frac{2}{5}$

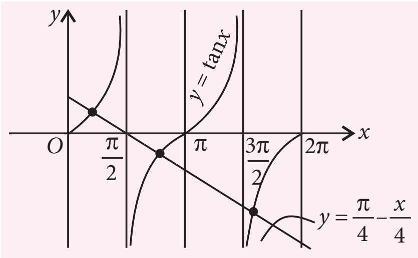
As $\theta\in[0, 2\pi]$, two solutions from I and II quadrants and two solutions from III and IV quadrants.

∴ Total number of solutions = 4

10. (a) : ∵ $x+2\tan x=\frac{\pi}{2} \Rightarrow \tan x=\frac{\pi}{4}-\frac{x}{2}$

If $y=\tan x=\frac{\pi}{4}-\frac{x}{2}$

$\Rightarrow \ y=\tan x$ ......(i) and $y=\frac{\pi}{4}-\frac{x}{2}$ ...(ii)



From the graph, (i) and (ii) intersect at 3 points

$\therefore$ Number of solutions = 3

11. (a, b, c, d) : $\because f_n(\alpha) = \tan(\alpha/2)\,(1 + \sec\alpha)\,(1 + \sec 2\alpha)$ $(1 + \sec 4\alpha)\ .....\ (1 + \sec 2^n\alpha)$

$$= \tan\left(\frac{\alpha}{2}\right)\left(\frac{1+\cos\alpha}{\cos\alpha}\right)(1+\sec 2\alpha)(1+\sec 4\alpha)....(1+\sec 2^n\alpha)$$

$$= \left(\frac{\sin(\alpha/2)}{\cos(\alpha/2)}\right)\left(\frac{2\cos^2(\alpha/2)}{\cos\alpha}\right)$$

$$(1 + \sec 2\alpha)(1+\sec 4\alpha).....(1+\sec 2^n\alpha)$$

$$= \tan\alpha\left(\frac{1+\cos 2\alpha}{\cos 2\alpha}\right)(1+\sec 4\alpha).....(1+\sec 2^n\alpha)$$

$= \tan 2\alpha(1 + \sec 4\alpha)\ ....(1 + \sec 2^n\alpha)$

$= \tan 4\alpha(1 + \sec 8\alpha)\ ....\ (1 + \sec 2^n\alpha)$

$= \tan 8\alpha(1 + \sec 16\alpha)\ ....(1 + \sec 2^n\alpha)$

$= \tan 2^n\alpha$ thus $f_n(\alpha) = \tan 2^n\alpha$

$$\therefore\ f_2\left(\frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right),\ f_3\left(\frac{\pi}{32}\right) = \tan\frac{\pi}{4},$$

$$f_4\left(\frac{\pi}{64}\right) = \tan\frac{\pi}{4},\ f_5\left(\frac{\pi}{128}\right) = \tan\frac{\pi}{4}$$

$$\therefore\ f_2\left(\frac{\pi}{16}\right) = f_3\left(\frac{\pi}{32}\right) = f_4\left(\frac{\pi}{64}\right) = f_5\left(\frac{\pi}{128}\right) = 1$$

(as $\tan\pi/4 = 1$)

12. (b, d) :

Given, $y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3 = (\sin^{-1}x + \cos^{-1}x)$ $\{(\sin^{-1}x)^2 - \sin^{-1}x\cos^{-1}x + (\cos^{-1}x)^2\}$

$$\therefore\ y = \frac{\pi}{2}\{\sin^{-1}x + \cos^{-1}x)^2 - 3\sin^{-1}x\cos^{-1}x\}$$

$$= \frac{\pi}{2}\left\{\frac{\pi^2}{4} - 3\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)\right\}$$

$$= \frac{\pi}{2}\left\{3(\sin^{-1}x)^2 - \frac{3\pi}{2}\sin^{-1}x + \frac{\pi^2}{4}\right\}$$

$$\Rightarrow\ 3(\sin^{-1}x)^2 - \frac{3\pi}{2}\sin^{-1}x + \left(\frac{\pi^2}{4} - \frac{2y}{\pi}\right) = 0,$$

For real value $D \ge 0 \Rightarrow \dfrac{9\pi^2}{4} - 4(3)\left(\dfrac{\pi^2}{4} - \dfrac{2y}{\pi}\right) \ge 0$

$$\Rightarrow\ \frac{9\pi^2}{4} - 3\pi^2 + \frac{24y}{\pi} \ge 0\ \Rightarrow\ \frac{24y}{\pi} \ge \frac{3\pi^2}{4}$$

$$\Rightarrow\ y \ge \frac{\pi^3}{32} \qquad ...(i)$$

Again $-1 \le \sin^{-1}x \le 1 \Rightarrow -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

$$\therefore\ y(-1) = \{(\sin^{-1}x)^3 + (\cos^{-1}x)^3\}_{x=-1}$$

$$= (\sin^{-1}(-1))^3 + (\cos^{-1}(-1))^3 = -\frac{\pi^3}{8} + \pi^3 = \frac{7\pi^3}{8} \qquad ...(ii)$$

Similarly, $y(1) = (\sin^{-1}(1))^3 + (\cos^{-1}1)^3$

$$= \frac{\pi^3}{8} + 0 = \frac{\pi^3}{8} \qquad ...(iii)$$

Form (i), (ii) and (iii) we have $\dfrac{\pi^3}{32} < \dfrac{\pi^3}{8} < \dfrac{7\pi^3}{8}$

$\therefore$ Least value of $y = \dfrac{\pi^3}{32}$ and greatest value of $y = \dfrac{7\pi^3}{8}$

13. (a, d) : $\because$ $x = a\sin^{1008}\alpha\cos^{1009}\alpha$ and $y = a\sin^{1009}\alpha\cos^{1008}\alpha$

$\therefore$ $x^2 + y^2 = a^2\sin^{2016}\alpha\cos^{2018}\alpha + a^2\sin^{2018}\alpha\cos^{2016}\alpha$

$= a^2\sin^{2016}\alpha\cos^{2016}\alpha$

and $xy = a^2\sin^{2017}\alpha\cos^{2017}\alpha = a^2(\sin\alpha\cos\alpha)^{2017}$

$$\therefore\ \frac{(x^2+y^2)^l}{(xy)^m} = \frac{a^{2l}(\sin\alpha\cos\alpha)^{2016l}}{a^{2m}(\sin\alpha\cos\alpha)^{2017m}}$$

which is independent of α and it will be possible only if $2016l = 2017\ m$

$\Rightarrow\ l = 2017,\ m = 2016$

14. (a, b, c, d) : $(1+ax)^n = 1 + nax + \dfrac{n(n-1)}{2!}a^2x^2 + ...$

$= 1 + 16x + 112x^2 + ...$

On equating the coefficients of x and x^2 we get

$na = 16$ and $\dfrac{n(n-1)}{2}a^2 = 112$

$\Rightarrow\ na(na - a) = 112 \times 2$

$\Rightarrow\ 16(16 - a) = 112 \times 2$

$\Rightarrow\ 16 - a = 14\ \Rightarrow\ a = 2\ \therefore\ n = 8$

Now $\tan^{-1}x + \cot^{-1}y = \tan^{-1}5$

$$\Rightarrow\ \tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}5 - \tan^{-1}x = \tan^{-1}\frac{5-x}{1+5x}$$

$$\Rightarrow\ \frac{1}{y} = \frac{5-x}{1+5x}\ \Rightarrow\ y = \frac{1+5x}{5-x} \qquad ....(*)$$

As x, y are positive integers

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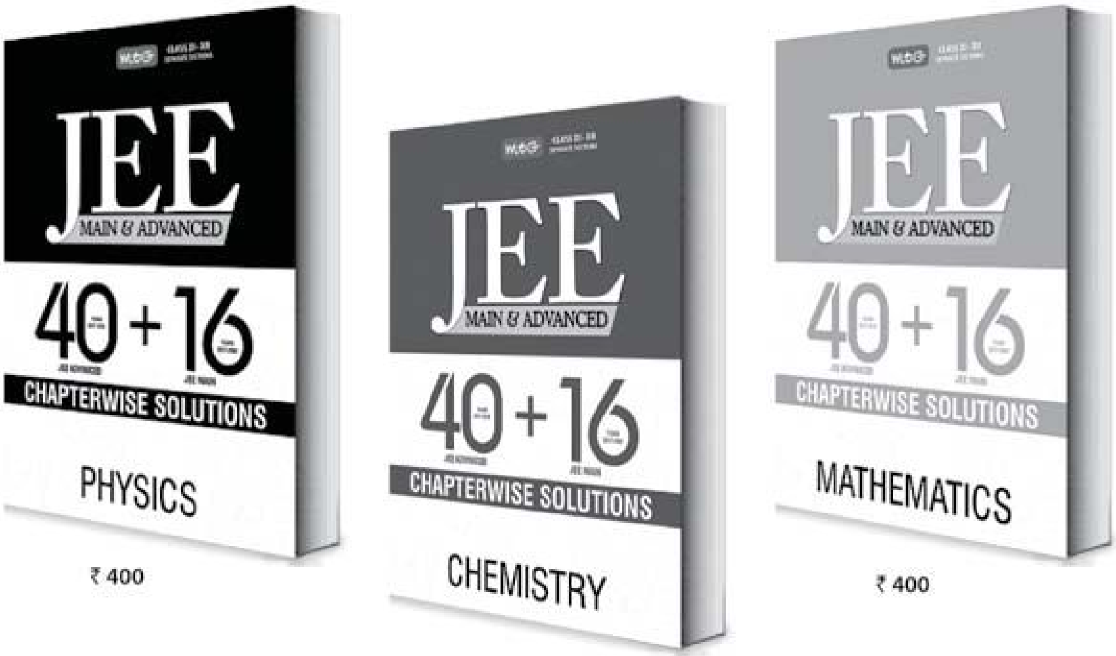

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$\therefore$ From equation (*) the possible value of x are 3, 4 when $x = 3$, $y = 8$ and when $x = 4$, $y = 21$

$\therefore \quad m = 2$

Also, $\dfrac{n}{ma} = \dfrac{8}{2\times 2} = 2$

15. (a, b) : $\because \; f(\alpha) = e^{\cos^{-1}\sin\left(\alpha+\frac{\pi}{3}\right)}$

$$\therefore \; f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)} = e^{\cos^{-1}\sin\left(\frac{9\pi}{18}+\frac{13\pi}{18}\right)} = e^{\cos^{-1}\sin\left(\frac{\pi}{2}+\frac{13\pi}{18}\right)}$$

$$= e^{\cos^{-1}\left(\cos\frac{13\pi}{18}\right)} = e^{\frac{13\pi}{18}}$$

$$\therefore \; f\left(-\frac{7\pi}{4}\right) = e^{\cos^{-1}\sin\left(-\frac{7\pi}{4}+\frac{\pi}{3}\right)}$$

$$= e^{\cos^{-1}\sin\left(-\frac{17\pi}{12}\right)} = e^{\cos^{-1}\left\{-\sin\left(\frac{3\pi}{2}-\frac{\pi}{12}\right)\right\}}$$

$$= e^{\cos^{-1}\left(\cos\frac{\pi}{12}\right)} = e^{\pi/12}$$

16. (a, b, c, d) : Given, $\dfrac{\tan 3A}{\tan A} = \lambda$

$$\Rightarrow \frac{\tan 3A - \tan A}{\tan A} = \frac{\lambda - 1}{1} \Rightarrow \frac{2\cos A}{\cos 3A} = \lambda - 1$$

$$\Rightarrow \frac{\cos A}{\cos 3A} = \frac{\lambda-1}{2} \qquad \ldots\text{(i)}$$

Also $\dfrac{\tan 3A}{\tan A} = \lambda$

$$\Rightarrow \frac{\sin 3A}{\cos 3A}\cdot\frac{\cos A}{\sin A} = \lambda \Rightarrow \frac{\sin 3A}{\sin A}\cdot\frac{\cos A}{\cos 3A} = \lambda,$$

on using (i) we get

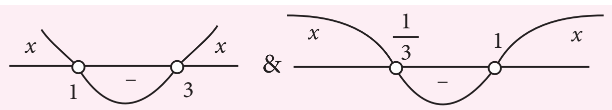
$$\frac{\sin 3A}{\sin A} = \frac{2\lambda}{\lambda - 1}$$

$$\Rightarrow \frac{3\sin A - 4\sin^3 A}{\sin A} = \frac{2\lambda}{\lambda-1} \Rightarrow 3 - 4\sin^2 A = \frac{2\lambda}{\lambda - 1}$$

or, $4\sin^2 A = \dfrac{\lambda-3}{\lambda-1}$ as $0 \le \sin^2\theta \le 1$

$$\Rightarrow 0 < \frac{\lambda-3}{\lambda-1} < 4 \text{ (as } \sin A \ne 0 \text{ or } 1)$$

$$\Rightarrow \frac{\lambda-3}{\lambda-1} > 0 \text{ and } \frac{\lambda-3}{\lambda-1} < 4 \Rightarrow \frac{3\lambda-1}{\lambda-1} > 0$$



$$\Rightarrow \lambda \in (-\infty, 1)\cup(3,\infty) \ldots\text{(ii)} \;\&\; \lambda \in \left(-4, \frac{1}{3}\right)\cup(1,\infty)\ldots\text{(iii)}$$

From (ii) and (iii) we get, $\lambda < 1/3$ and $\lambda > 3$.

17. (a, c) : Given, $x + y = \dfrac{\pi}{2}$ ...(i)

and $\sin x + \sin y = \sqrt{2}$

$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \sqrt{2}$$

$$\Rightarrow \cos\left(\frac{x-y}{2}\right) = 1 \qquad \text{(using (i))}$$

$$\Rightarrow x - y = 2\cdot 2k\pi \Rightarrow x - y = 4k\pi,\; k \in I \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$x = 2k\pi + \frac{\pi}{4} \text{ and } y = \frac{\pi}{4} - 2k\pi, k \in I$$

18. (c) **19. (b)** **20. (d)**

21. (a) : $\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + \cot^{-1}21 + \cot^{-1}31 + \ldots..$

We can write the general form of the series 3, 7, 13, 21, 31, ... by successive order of difference,

$S_n = 3 + 7 + 13 + 21 + 31 + \ldots. + t_n$

$S_n = 3 + 7 + 13 + \ldots. + t_{n-1} + t_n$

On subtracting we get

$t_n = 3 + (4 + 6 + \ldots. + t_{n-1}) = 3 + (n - 1)(n + 2)$

$t_n = 1 + n + n^2$

Now, $\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + \ldots..$ up to infinite terms

$$= \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}(1+r+r^2) = \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

$$= \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{(r+1)-r}{1+r(r+1)}\right)$$

$$= \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}[\tan^{-1}(r+1) - \tan^{-1}r]$$

$$= \operatorname*{Lt}_{n\to\infty}[\tan^{-1}(n+1) - \tan^{-1}1] = \tan^{-1}\infty - \tan^{-1}1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

22. (d) : Given series is

$$\tan^{-1}\left(\frac{2}{2+1^2+1^4}\right) + \tan^{-1}\left(\frac{4}{2+2^2+2^4}\right) + \tan^{-1}\left(\frac{6}{2+3^2+3^4}\right) + \ldots..\infty$$

$$= \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)$$

$$= \operatorname*{Lt}_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left[\frac{(r^2+r+1)-(r^2-r+1)}{1+(r^2+r+1)(r^2-r+1)}\right]$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}[\tan^{-1}(r^2+r+1)-\tan^{-1}(r^2-r+1)]$$

$$= \lim_{n\to\infty} (\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+(\tan^{-1}13-\tan^{-1}7) + ...+ (\tan^{-1}(n^2+n+1)-\tan^{-1}(n^2-n+1))$$

$$= \lim_{n\to\infty} [\tan^{-1}(n^2+n+1)-\tan^{-1}1]$$

$$= \tan^{-1}\infty-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

or, $\lim_{n\to\infty} \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)= \lim_{n\to\infty} \tan^{-1}\left(\frac{1+1/n}{\frac{2}{n}+1+\frac{1}{n}}\right)$

$$=\tan^{-1}1=\frac{\pi}{4}$$

23. (b) : Given infinite series is

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{4}{33}\right)+\tan^{-1}\left(\frac{8}{129}\right)+...\infty \quad ...(*)$$

$\therefore$ General term of (*) is $\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$

$\therefore$ Sum of $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{4}{33}\right)$

$$+\tan^{-1}\left(\frac{8}{129}\right)+.....\infty = \lim_{n\to\infty} \sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^r\cdot 2^{r-1}}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r\cdot 2^{r-1}}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n}[\tan^{-1}(2^r)-\tan^{-1}(2^{r-1})]$$

$$= \lim_{n\to\infty} (\tan^{-1}2^n-\tan^{-1}2^{\circ})$$

(by putting r = 1, 2, ... n and simplifying)

$$=\tan^{-1}2^{\infty}-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

24. (d) **25. (c)** **26. (b)**

27. A → 4, B → 3, C → 2, D → 1

A. $\alpha = 6° \therefore 5\alpha=30^{\circ} \Rightarrow \cos 5\alpha=\cos 30^{\circ}=\frac{\sqrt{3}}{2}$

$\therefore$ $\cos5\alpha = \cos(2\alpha + 3\alpha) = \cos2\alpha \cos3\alpha - \sin2\alpha \sin3\alpha$

$$\Rightarrow \cos 30°=16\cos^5\alpha-20\cos^3\alpha+5\cos\alpha$$

$$\Rightarrow \frac{\sqrt{3}}{2}=16x^5-20x^3+5x$$

$$\Rightarrow 32x^5-40x^3+10x-\sqrt{3}=0 \text{ where } x=\cos\alpha=\cos 6°$$

B. Let $\alpha=10° \Rightarrow 6\alpha=60°$

$$\therefore \cos 6\alpha=\frac{1}{2} \text{ or } \cos 3(2\alpha)=\frac{1}{2}$$

$$\Rightarrow 4\cos^3 2\alpha-3\cos 2\alpha=\frac{1}{2}$$

$$\Rightarrow 4(2\cos^2\alpha-1)^3-3(2\cos^2\alpha-1)=\frac{1}{2}$$

$$\Rightarrow 8(2\cos^2\alpha-1)^3-6(2\cos^2\alpha-1)-1=0$$

$$\Rightarrow 8\left(2\cdot\frac{x^2}{4}-1\right)^3-6\left(2\cdot\frac{x^2}{4}-1\right)=1$$

(By putting $2\cos\alpha = x$)

$$\Rightarrow (x^2-2)^3-3(x^2-2)=1$$

$$\Rightarrow x^6-6x^4+9x^2-3=0 \text{ where } x=2\cos 10°$$

C. Let α = 15° $\therefore$ 3α = 45°

$\therefore$ $\tan3\alpha = \tan45° = 1$

$$\Rightarrow 3\tan\alpha-\tan^3\alpha=1-3\tan^2\alpha$$

$$\Rightarrow \tan^3\alpha-3\tan^2\alpha-3\tan\alpha+1=0$$

$$\Rightarrow x^3-3x^2-3x+1=0 \text{ where } x=\tan\alpha=\tan 15°$$

D. Let $\alpha=20° \therefore 3\alpha=60° \Rightarrow \cos 3\alpha=\cos 60°=\frac{1}{2}$

$$\Rightarrow \cos 3\alpha=\frac{1}{2}$$

$$\Rightarrow 4\cos^3\alpha-3\cos\alpha=\frac{1}{2} \text{ or } 8\cos^3\alpha-6\cos\alpha-1=0$$

or $8x^3 - 6x - 1 = 0$ where $x = \cos20°$

28. A → 3, B → 1, C → 4, D → 2

A. $\cos^{-1}\left(\frac{3}{5}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{2/3}{1-\frac{1}{9}}\right)$

$$=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)$$

$$\therefore \cos^{-1}\left(\frac{3}{5}\right)+2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{3}{4}\right)$$

$$=\tan^{-1}\left(\frac{4}{3}\right)+\cot^{-1}\left(\frac{4}{3}\right)=\frac{\pi}{2}$$

$$\left(\text{Using } \tan^{-1}\alpha+\cot^{-1}\alpha=\frac{\pi}{2}\right)$$

B. $\cos^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)$

$$=\tan^{-1}\left(\frac{12}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{63}{16}\right)$$

$$=\left[\pi+\tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{36}{20}}\right)\right]+\tan^{-1}\left(\frac{63}{16}\right)$$

$$\left(\text{using } \tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right.$$
$$\left.\text{if } x, y>0 \text{ and } xy>1\right)$$

$$=\pi+\tan^{-1}\left(\frac{63}{-16}\right)+\tan^{-1}\left(\frac{63}{16}\right)$$

$$=\pi-\tan^{-1}\left(\frac{63}{16}\right)+\tan^{-1}\left(\frac{63}{16}\right)=\pi$$

C. $\sin^{-1}\left(\frac{1}{5\sqrt{2}}\right)+2\tan^{-1}\left(\frac{1}{3}\right)$

$$=\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{3}{4}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}\right)$$

$$=\tan^{-1}\left(\frac{25}{25}\right)$$

$$=\tan^{-1}(1)=\tan^{-1}\tan\left(\frac{\pi}{4}\right)=\frac{\pi}{4}$$

D. $\therefore\ A=\tan^{-1}\left(\frac{x\sqrt{3}}{2\lambda-x}\right), B=\tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$

$$\therefore\ \tan(A-B)=\frac{\frac{x\sqrt{3}}{2\lambda-x}-\frac{(2x-\lambda)}{\lambda\sqrt{3}}}{1+\frac{x\sqrt{3}}{2\lambda-x}\cdot\frac{2x-\lambda}{\lambda\sqrt{3}}}$$

$$\frac{3\lambda x+(2x-\lambda)(x-2\lambda)}{\sqrt{3}[\lambda(2\lambda-x)+x(2x-\lambda)]}$$

$$=\frac{1}{\sqrt{3}}\left(\frac{2x^2-(5\lambda x-3\lambda x)+2\lambda^2}{2x^2-2\lambda x+2\lambda^2}\right)=\frac{1}{\sqrt{3}}$$

$$\therefore\ \tan(A-B)=\tan\frac{\pi}{6} \quad\Rightarrow\quad A-B=\frac{\pi}{6}$$

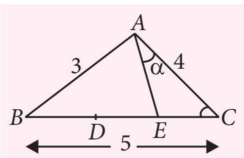
29. (3) : As the side of ΔABC are $AB = 3$, $BC = 5$, $CA = 4$

$\therefore$ Triangle ABC is right angle Δ at A, *i.e.*, $\angle BAC = 90°$

In ΔCAE, let $\angle CAE=\alpha$ and $CE=\frac{5}{3}$

$$\text{we have } \cos C=\frac{4^2+\left(\frac{5}{3}\right)^2-(AE)^2}{2\cdot 4\cdot\left(\frac{5}{3}\right)}=\frac{16+\frac{25}{9}-(AE)^2}{\frac{40}{3}}$$

$$\therefore\ \frac{4}{5}=\frac{\frac{169}{9}-(AE)^2}{\frac{40}{3}}$$



$$\Rightarrow\ \frac{169}{9}-(AE)^2=\frac{4\times 40}{5\times 3}=\frac{32}{3}$$

$$\Rightarrow\ (AE)^2=\frac{169}{9}-\frac{32}{3}=\frac{169-96}{9}=\frac{73}{9}\ \therefore\ AE=\frac{\sqrt{73}}{3}$$

Again in ΔCAE we have

$$\cos\alpha=\frac{AC^2+(AE)^2-\left(\frac{5}{3}\right)^2}{2\cdot 4AC}=\frac{16+\frac{73}{9}-\frac{25}{9}}{8\times\frac{\sqrt{73}}{3}}$$

$$\Rightarrow\ \cos\alpha=\frac{144+73-25}{9}\times\frac{3}{8\sqrt{73}}=\frac{8}{\sqrt{73}}\Rightarrow\sec\alpha=\frac{\sqrt{73}}{8}$$

$$\text{Now } \tan^2\alpha=\sec^2\alpha-1=\frac{73}{64}-1=\frac{9}{64}$$

$$\therefore\ \tan\alpha=\frac{3}{8}\ \textit{i.e.}, \tan\angle CAE=\frac{3}{8}$$

$\Rightarrow$ $8\tan\angle CAE = 3$

30. (3) : Let $ABCD$ be the cyclic quadrilateral in which $AB = 2$ and $BC = 5$ and $\angle ABC = 60°$

$\therefore$ $\angle ADC = 180° - 60° = 120°$

$\angle ABC=60°\ \therefore\ \angle ADC=180°-60°=120°$

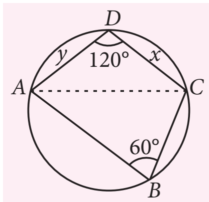
Area of quadrilateral $ABCD$ = $ar(\Delta ABC)$ + $ar(\Delta ACD)$

$$\Rightarrow\ \frac{1}{2}AB\times BC\sin 60°+\frac{1}{2}CD\times DA\sin 120°=4\sqrt{3}$$

$$\Rightarrow\ \frac{1}{2}\times 2\times 5\times\frac{\sqrt{3}}{2}+\frac{1}{2}xy\left(\frac{\sqrt{3}}{2}\right)=4\sqrt{3}$$

where $CD = x$, $AD = y$

$\Rightarrow xy=6$ ....(i)



From ΔABC we get

$$\cos 60°=\frac{(AB)^2+(BC)^2-(AC)^2}{2AB\cdot BC}$$

$$\Rightarrow\ (AC)^2=(AB)^2+(BC)^2-2(AB)(BC)\left(\frac{1}{2}\right)$$

$= 4 + 25 - 2 \times 5\ \therefore\ (AC)^2 = 19$ ...(ii)

Again from ΔACD, we get

$(AC)^2 = (CD)^2 + (DA)^2 - 2CD \times DA\cos 120°$

$$(AC)^2=x^2+y^2-2xy\left(-\frac{1}{2}\right)=x^2+y^2+xy$$

$= x^2 + y^2 + 6$ (using (i)) ...(iii)

From (ii) and (iii) we have

$x^2+y^2+6=19\ \Rightarrow\ x^2+y^2=13$

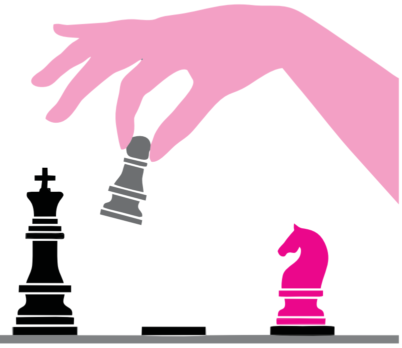
$\Rightarrow\ x=2$ then $y=3$ and $x=3$ then $y=2$

$\therefore$ $x = 2, 3$ and $y = 3, 2$

$\therefore$ Length of larger side of the quadrilateral from the rest sides is 3

31. (1)

Challenging PROBLEMS ON Vectors and 3D Geometry



1. A plane π is given and on one side of the plane three non-collinear points A, B, C such that the plane determined by them is not parallel to π. Three arbitrary points A', B', C' in π are selected. Let L, M, N be the midpoints of AA', BB' and CC' and let G be the centroid of ΔLMN. The locus of G as A', B', C' varies across π is
(a) a plane parallel to π
(b) a plane perpendicular to π
(c) a line parallel to π
(d) a line perpendicular to π

2. Consider the two statements regarding a tetrahedron $ABCS$
S_1 : $ABCS$ has 5 different spheres that touch all 6 lines determined by its edges.
S_2 : $ABCS$ is a regular tetrahedron. Then
(a) $S_1 \Rightarrow S_2$ (b) $S_2 \Rightarrow S_1$
(c) $S_1 \Leftrightarrow S_2$ (d) $S_2 \nRightarrow S_1$

3. If all the angles of a convex n-gon are equal and the lengths of consecutive edges $a_1, a_2, \ldots, a_n$ satisfy $a_1 \geq a_2 \geq \ldots \geq a_n$ then
(a) $a_1 = a_n$ (b) $a_2 = 2a_{n-1}$
(c) $a_3 = a_{n-2} + a_{n-1}$ (d) $a_3 = a_{n-2} - a_{n-1}$

4. Consider a tetrahedron $ABCD$, with O be the centre of the circumsphere of regular tetrahedron $ABCD$ and let P be any point. Let $T = OA + OB + OC + OD$ and $Y = PA + PB + PC + PD$, then
(a) $T > Y$ (b) $T < Y$
(c) $T = Y$ (d) $T = 2Y$

5. Exactly one side of a tetrahedron is of length greater than 1, then max. volume of the tetrahedron is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

6. In any tetrahedron there is a vertex such that the lengths of its sides through that vertex are sides of a triangle. This statement is
(a) always true
(b) always false
(c) true only in regular tetrahedrons
(d) not true in general

7. The condition on 'a' such that there exists a tetrahedron of whose one edge has length 'a' and the other 5 edges have length 1 is
(a) $a < 2$ (b) $a < \sqrt{3}$
(c) $a < 2\sqrt{2}$ (d) $a < 2\sqrt{3}$

8. Vertex A of the acute triangle ABC is equidistant from the circumcentre O and orthocentre H. The possible value of $\angle A$ is
(a) 30° (b) 60° (c) 75° (d) 90°

9. The lengths of two opposite edges of a tetrahedron are 12 and 15 units, their shortest distance is 10 units. If the volume of the tetrahedron is 200 cubic units then the angle between the two edge is
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$
(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\sin^{-1}\left(\frac{4}{5}\right)$

10. The plane $lx + my + nz = p$ intersects co-ordinate axes in A, B and C respectively. If area of ΔABC is Δ then
(a) $p^2 \leq \frac{2\Delta}{3\sqrt{3}}$ (b) $p^2 \geq \frac{2\Delta}{3\sqrt{3}}$
(c) $p^2 > \frac{2\Delta}{3\sqrt{3}}$ (d) $p^2 < \frac{2\Delta}{3\sqrt{3}}$

11. $\vec{a}, \vec{b}, \vec{c}$ are three non-zero non-coplanar vectors with fixed magnitudes. Angles between any of the vector with the normal of the plane containing other two is α. Volume is T and surface area is Y, then $\frac{2}{|\cos\alpha|} \cdot \left[\frac{1}{|\vec{a}|} + \frac{1}{|\vec{b}|} + \frac{1}{|\vec{c}|}\right] =$

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(a) $\frac{Y}{T}$ (b) $\frac{T}{Y}$ (c) $\frac{2T}{Y}$ (d) $\frac{2Y}{T}$

12. The shortest distance between the lines $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ and $x = y = z$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{4}}$ (d) $\frac{1}{\sqrt{5}}$

13. Consider the four coplanar points $A(\vec{O}), B(3\hat{i}+f(t)\hat{j}+\hat{k}), C(2\hat{i}+f'(t)\hat{j}+2\hat{k})$ and $D(\hat{i}+\hat{j})$, then $f(x)$ is of the form

(a) $2 + \lambda e^{2x}$ (b) $2 + \lambda\log(2x)$

(c) λe^x (d) $\lambda\log x$

14. Let $\vec{a}$ be a vector or rectangular co-ordinate system with sloping angle 60°. Suppose that $|\vec{a}-\hat{i}|$ is geometric mean of $|\vec{a}|$ and $|\vec{a}-2\hat{i}|$, then $|\vec{a}|=$

(a) $\sqrt{2}$ (b) $\sqrt{2}+1$

(c) $\sqrt{2}-1$ (d) 2

15. The volume of the solid enclosed by the planes $|x| + |y| + |z| = 1$ is

(a) $\frac{1}{3}$ cubic units (b) $\frac{2}{3}$ cubic units

(c) $\frac{3}{3}$ cubic units (d) $\frac{4}{3}$ cubic units

SOLUTIONS

1. (a) : Let $d(P)$ denote the distance of a point P from π, with P being on the same side of π as A, B, C. Let G_1 be the fixed centroid of ΔABC and G_1' be the centroid of $\Delta A'B'C'$. It is easy to prove that G is the midpoint of G_1G_1'. Hence, varying G_1' across π, we have the locus of G as the plane π' parallel to π such that

$$P \in \pi' \Rightarrow d(P) = \frac{d(G_1)}{2} = \frac{d(A)+d(B)+d(C)}{6}$$

2. (c) : The spheres are arranged in similar way, where we have one in-circle and 3 ex-circles. Here, we have one in-sphere and 4 ex-spheres. For the in-sphere, we have $SA + BC = SB + CA = SC + AB$ and for the ex-sphere opposite S, we have $SA - BC = SB - CA = SC - AB$. Hence $SA = SB = SC$ and $AB = BC = CA$. By symmetry, $AB = AC = SA$, *i.e.* regular tetrahedron.

3. (a) : Let $\overrightarrow{OA_1}, \overrightarrow{OA_2}, ..., \overrightarrow{OA_n}$ be the vectors corresponding to edges $a_1, a_2, ..., a_n$ of the polygon. By the conditions in the question,

$$\overrightarrow{OA_1} + \overrightarrow{OA_2} + ... + \overrightarrow{OA_n} = \vec{O},$$

$$\angle A_1OA_2 = \angle A_2OA_3 = ... = \frac{2\pi}{n}$$

and $\overrightarrow{OA_1} \geq \overrightarrow{OA_2} \geq \overrightarrow{OA_3} \geq .... \geq \overrightarrow{OA_n}$.

Let l be a line through O, perpendicular to OA_n and $B_1, B_2, ..., B_{n-1}$ be the projections of $A_1, A_2, ..., A_{n-1}$ on l.

By the assumptions, $\sum \overrightarrow{OB_i} = \vec{O}$.

But, $\overrightarrow{OB_i} \leq \overrightarrow{OB_{n-i}}$ for all $i \leq \frac{n}{2}$,

because all the sums $\overrightarrow{OB_i} + \overrightarrow{OB_{n-i}}$ lies on the same side of O. Hence, all these sums must be $\vec{O}$.

i.e. $OA_i = OA_{n-i}$.

4. (b) : Let $O(0, 0, 0)$, $A(-a, -a, -a)$, $B(-a, a, a)$, $C(a, -a, a)$ and $D(a, a, -a)$. For a point $P(x, y, z)$ by using RMS $\geq$ AM.

We have, $PA + PB + PC + PD$ does not exceed

$$2\sqrt{PA^2 + PB^2 + PC^2 + PD^2}.$$

Now, $PA^2 + PB^2 + PC^2 + PD^2$

$= 4(x^2 + y^2 + z^2) + 12a^2$

$\geq 12a^2 = OA^2 + OB^2 + OC^2 + OD^2$

Hence, $PA + PB + PC + PD$

$\geq 2\sqrt{OA^2 + OB^2 + OC^2 + OD^2}$

$= OA + OB + OC + OD$

5. (d) : Suppose CD is the longest edge of the tetrahedron $ABCD$. Let $AB = a$, CK and DL be the altitudes of ΔABC and ΔABD respectively. Let DM be the altitude of tetrahedron.

Then in right triangle AKC, $CK^2 \leq 1 - \frac{a^2}{4}$.

Similarly, $DL^2 \leq 1 - \frac{a^2}{4}$.

Since $DM \leq DL$, so $DM^2 \leq 1 - \frac{a^2}{4}$

Hence, volume $V = \frac{1}{3}\cdot[DM]\left[\frac{1}{2}\cdot a\cdot CK\right] \leq \frac{1}{6}\cdot a\cdot\left(1-\frac{a^2}{4}\right)$

$$\leq \frac{1}{24}(2+a)[1-(a-1)^2] \leq \frac{1}{24}\cdot 3\cdot 1 = \frac{1}{8}$$

6. (a) : Let AB be the largest edge of the tetrahedron $ABCD$. Then $AB \geq AC + AD$ and $AB \geq BC + BD$.

$\Rightarrow 2AB \geq AC + AD + BC + BD$

$\Rightarrow$ either $AB \geq AC + BC$ or $AB \geq AD + BD$

Contradicting the triangle inequality. Hence, the three edges coming out of atleast one of the vertices A and B form a triangle.


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7. (b) : Let $AB = a$ and remaining edges have length 1. Let M be the midpoint of CD. Then since, CDA and BCD are equilateral triangles, we have $AM = BM$ $= \sqrt{3}/2$ and so $AB < AM + BM$ gives $a < \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

i.e., $a < \sqrt{3}$

8. (b) : Let origin be the circumcentre, then according to question,

$|\vec{A} - \vec{0}| = |\vec{A}| = |\vec{A} - \vec{H}|$ and

circumradius, $R = |\vec{A}| = |\vec{B}| = |\vec{C}|$ and $\vec{H} = \vec{A} + \vec{B} + \vec{C}$.

Let $a = |\vec{B} - \vec{C}|$, then $R^2 = |\vec{A}|^2 = |\vec{A} - \vec{H}|^2 = |\vec{B} + \vec{C}|^2$

So, $R^2 = 4R^2 - a^2$ i.e. $\frac{a}{R} = \sqrt{3} \Rightarrow \sin A = \frac{\sqrt{3}}{2}$

by using sine rule.

Hence, $\angle A = 60°$

9. (b) : Use, volume $= \frac{1}{6} abd \sin\theta$

10. (a) : $\Delta^2 = \Delta_x^2 + \Delta_y^2 + \Delta_z^2$

$$= \frac{1}{4} \cdot \left[\left(\frac{p^2}{lm}\right)^2 + \left(\frac{p^2}{mn}\right)^2 + \left(\frac{p^2}{ln}\right)^2 \right] = \frac{p^4}{4l^2m^2n^2}$$

i.e. $\Delta = \frac{p^2}{2lmn}$.

Now, A.M. $\geq$ G.M. gives

$\frac{l^2 + m^2 + n^2}{3} \geq (lmn)^{2/3}$ i.e. $\frac{1}{3\sqrt{3}} \geq \frac{p^2}{2\Delta}$

Hence, $p^2 \leq \frac{2\Delta}{3\sqrt{3}}$, with equality occurring at $l = m = n$.

11. (a) : $T = |\vec{a} \cdot (\vec{b} \times \vec{c})| = |\vec{b} \cdot (\vec{c} \times \vec{a})| = |\vec{c} \cdot (\vec{a} \times \vec{b})|$

$T = |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_1|$

$= |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_2| = |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_3|$

and surface area $Y = 2(|\vec{a} \times \vec{b}| + |\vec{b} \times \vec{c}| + |\vec{c} \times \vec{a}|)$

i.e., $Y = 2(|\vec{a}||\vec{b}||\sin\theta_1| + |\vec{b}||\vec{c}||\sin\theta_2| + |\vec{c}||\vec{a}||\sin\theta_3|)$

12. (a) : Any plane passing through the first line can be taken as $2x + y + z - 1 + \lambda(3x + y + 2z - 2) = 0$. If this is parallel to the second line then

$(2 + 3\lambda) \cdot 1 + (1 + \lambda) \cdot 1 + (1 + 2\lambda) \cdot 1 = 0$

$\Rightarrow \lambda = \frac{-2}{3}$

So, plane is $y - z + 1 = 0$.

Distance from (0, 0, 0) is $\frac{1}{\sqrt{2}}$.

13. (a) : As A, B, C, D are coplanar $\therefore [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

Simplifying this gives, $f'(t) - 2f(t) = -4$

i.e., $f(x) = 2 + \lambda e^{2x}$, $\lambda \in R$.

14. (c) : Let $\vec{a} = x\hat{i} + \sqrt{3}x\hat{j}$.

$|\vec{a}| = 2x, x > 0$

Now A.T.Q., $|\vec{a}||\vec{a} - 2\hat{i}| = |\vec{a} - \hat{i}|^2$

$\Rightarrow 2x \cdot \sqrt{(x-2)^2 + 3x^2} = (x-1)^2 + (3x^2)$

Simplifying $x = \frac{1}{2}(\sqrt{2} - 1)$

15. (d) : Let $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$, and $O(0, 0, 0)$

Reqd. volume = 8 × vol. of tetrahedran $OABC$

$= 8 \times \frac{1}{6}[\overline{OA}\ \overline{OB}\ \overline{OC}]$

where, $\overline{OA} = \hat{i}, \overline{OB} = \hat{j}, \overline{OC} = \hat{k}$

Hence, reqd. vol. $= \frac{4}{3}[\hat{i}\ \hat{j}\ \hat{k}] = \frac{4}{3}$ cubic units

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JEE Main 2018

MOCK TEST PAPER

Series-4

Time: 1 hr 15 min.

The entire syllabus of Mathematics of JEE MAIN is being divided into eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No. 4	Topic	Syllabus In Detail
	Mathematical Induction	Principle of mathematical induction and its simple applications
	Binomial Theorem & it's Simple Applications	Binomial Theorem of positive integral index, general term and middle term, properties of Binomial coefficients and simple applications
	Sequences and Series	Definition of series and sequence. Arithmetic, geometric and harmonic progressions. Arithmetic, geometric & harmonic means between two given numbers, relation between A.M.,G.M. and H.M. Sum up to n terms of special series: $\sum n, \sum n^2, \sum n^3$. Arithmetic-Geometric progression.
	Co-ordinate geometry-2D	Conic Sections: Sections of cones, equation conic sections(parabola, ellipse and hyperbola in standard forms, condition for y = mx + c to be a tangent and point(s) of tangency

1. Let $P(n): 1+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}$ is true for
   (a) $\forall n$ (b) for $n = 1$
   (c) $n > 1\ \forall\ n \in N$ (d) none of these
2. Let $P(n)$ be the statement represent the sum of three successive natural numbers $\forall n \in N$, then the smallest value of n to which $P(n)$ is divisible by 9 is
   (a) 1 (b) 3! (c) 3 (d) 9!
3. Let $P(n) = 5^n - 2^n$, $P(n)$ is divisible by 3λ where λ and n both are odd positive integers then the least value of n and λ will be
   (a) 13 (b) 11 (c) 1 (d) 5
4. Middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^{2n}$ is
   (a) $\frac{6n!}{3n!\,3n!}x^n$ (b) $\frac{6n!}{3n!}x^{3n}$
   (c) $\frac{6n!}{3n!\,3n!}(-x)^{3n}$ (d) none of these
5. If $(1 + x + x^2)^n = b_0 + b_1x + b_2x^2 + \ldots + b_{2n}x^{2n}$, then $b_0 + b_2 + b_4 + \ldots + b_{2n}$ equals
   (a) $\frac{3^n-1}{2}$ (b) $3^n+\frac{1}{2}$
   (c) $\frac{3^n+1}{2}$ (d) none of these.
6. Number of irrational terms in the expansion of $\left(\sqrt[5]{2}+\sqrt[10]{3}\right)^{60}$ are
   (a) 54 (b) 61 (c) 30 (d) 31
7. The range of the value of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha\in[-1,1]$ is
   (a) [1, 2] (b) (1, 2)
   (c) $\left(\frac{{}^{10}C_5\,\pi^2}{2^{20}}, \frac{-{}^{10}C_5\,\pi^2}{2^5}\right)$

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(d) $\left(\dfrac{-{}^{10}C_5\,\pi^{10}}{2^5}, \dfrac{{}^{10}C_5\,\pi^{10}}{2^{20}}\right)$

8. If C_0, C_1, C_2, C_3, ..., C_n denotes the binomial coefficients in the expansion of $(1 + x)^n$, then $\dfrac{C_1}{C_0}+2\dfrac{C_2}{C_1}+3\dfrac{C_3}{C_2}+...+n\dfrac{C_n}{C_{n-1}}=$

(a) $\dfrac{n}{2}$ (b) $\dfrac{n+1}{2}$ (c) $\dfrac{n(n-1)}{2}$ (d) $\dfrac{n(n+1)}{2}$

9. If the third term in the expansion of $\left(\dfrac{1}{x}+x^{\log_{10}x}\right)^5$ is 1,000, then $x =$

(a) 10^2 (b) 10^3
(c) 10 (d) none of these

10. The greatest term in the expansion of $(2x - 3y)^{28}$ when $x = 9$, $y = 4$ is

(a) 14^{th} (b) 15^{th}
(c) $14^{th}, 15^{th}$ (d) 12^{th}

11. If the sum of n terms of an A.P. is $cn\ (n - 1)$ where $n \neq 0$, then the sum of the squares of these terms is

(a) $c^2n^2(n + 1)^2$ (b) $\dfrac{2}{3}c^2n(n-1)(2n-1)$
(c) $\dfrac{2}{3}c^2n(n+1)(2n+1)$ (d) $\dfrac{1}{3}c^2n^2$

12. The interior angles of a polygon are in A.P. with the smallest angle and common difference being respectively 120° and 5°. Then $n =$

(a) 16 (b) 7 (c) 9 (d) 10

13. Between the numbers 1 and 31, m means are inserted so that the ratio of 7^{th} and the $(m - 1)^{th}$ mean is 5 : 9, then the value of m is

(a) 14 (b) 10 (c) 7 (d) 3

14. If the arithmetic mean of two positive numbers a and b $(a > b)$ is twice their G.M., then $a : b =$

(a) $6+\sqrt{7}:6-\sqrt{7}$ (b) $2+\sqrt{3}:2-\sqrt{3}$
(c) $5+\sqrt{6}:5-\sqrt{6}$ (d) none of these

15. Let $P=\dfrac{3}{17}+\dfrac{33}{17^2}+\dfrac{333}{17^3}+...\infty$ then P equals

(a) $\dfrac{3}{17}$ (b) $\dfrac{7}{17}$ (c) $\dfrac{3}{7}$ (d) $\dfrac{51}{112}$

16. Between the numbers 2 and 20, 8 A.M's are inserted, then their sum is

(a) 88 (b) 44
(c) 176 (d) none of these

17. If the roots of the equation $ax^3 + 3x^2 + cx + d = 0$ are in G.P., then

(a) $a^3c = bd^3$ (b) $ab^3 = cd^3$
(c) $a^3b = c^3d$ (d) $c^3a = b^3d$

18. The age of the father of two children is twice of the elder one added to four times that of the younger one. If the geometric mean of the ages of the two children is $4\sqrt{3}$ and their harmonic mean is 6, father's age (in years) is

(a) 36 (b) 40 (c) 50 (d) 56.

19. Let a_1, a_2, a_3, ... be a sequence such that $a_1 = 2$ and $a_n - a_{n-1} = 2n\ \forall\ n \geq 2$, then $a_1 + a_2 + ... + a_{20} =$

(a) 3040 (b) 3020 (c) 2020 (d) 3080

20. If three successive terms of a G.P. with common ratio $r(r > 1)$ form the sides of a ΔABC and $[r]$ denotes greatest integer function, then $[r] + [-r] =$

(a) 0 (b) 1
(c) –1 (d) none of these

21. The set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 16$ touches the parabola $y^2 = 8x$ is

(a) $(-\infty,-1)\cup(1,\infty)$ (b) $(-\infty,\infty)$
(c) $\left(-\infty,-\sqrt{\dfrac{\sqrt{2}-1}{2}}\right)\cup\left(\sqrt{\dfrac{\sqrt{2}-1}{2}},\infty\right)$
(d) none of these

22. If the tangent drawn at a point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as normal drawn at $(\sqrt{5}\cos\alpha, 2\sin\alpha)$ on the ellipse $\dfrac{x^2}{5}+\dfrac{y^2}{4}=1$, then which of the following is not true ?

(a) $t=\pm\dfrac{1}{\sqrt{5}}$ (b) $\alpha=-\tan^{-1}2$
(c) $\alpha=\tan^{-1}2$ (d) none of these

23. If two points are taken on minor axis of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at the same distance from the centre as the foci, the sum of the squares of the perpendiculars from these point on any tangent to the ellipse if $a < b$ is

(a) a^2 (b) b^2 (c) $2a^2$ (d) $2b^2$

24. The equation of parabola whose latus rectum is 2 units, axis of line is $x + y - 2 = 0$ and tangent at the vertex is $x - y + 4 = 0$

(a) $(x+y-2)^2=4\sqrt{2}(x-y+4)^2$

(b) $(x-y-4)^2 = 4\sqrt{2}(x+y-2)$

(c) $(x+y-2)^2 = 2\sqrt{2}(x-y+4)$

(d) none of these

25. If ASB is the focal chord of the parabola $y^2 = 8x$ such that $AS = 4$, then the length $SB =$

(a) 6 (b) 3

(c) 4 (d) none of these

26. Let $y = 4x^2$ and $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ intersect iff

(a) $|a| \le \frac{1}{\sqrt{2}}$ (b) $a < -\frac{1}{\sqrt{2}}$

(c) $a > -\frac{1}{\sqrt{2}}$ (d) none of these

27. The shortest distance between the parabola's $y^2 = 4x$ and $y^2 - 2x + 6 = 0$ is

(a) $\sqrt{2}$ (b) $\sqrt{5}$

(c) $\sqrt{3}$ (d) none of these.

28. If chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{52} + \frac{y^2}{13} = 1$ are at right angle then ratio of the product of abscissa and ordinate is

(a) $-16 : 1$ (b) $4 : 1$

(c) $16 : 1$ (d) none of these

29. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major and minor axes at A and B.If O is the origin, then the area of the ΔOAB (in sq. units) is

(a) 48 (b) 9 (c) 24 (d) 16

30. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$

(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

SOLUTIONS

1. (c) : For $n = 1$, L.H.S of $P(1) = 1$ and R.H.S. of $P(1) = 2 - 1 = 1$

$\therefore$ $P(1)$ is not true, as L.H.S of $P(1) = 1 < 1 =$ RHS. of $P(1)$, False

Again $n = 2$, L.H.S. of $P(2) = 1 + \frac{1}{4} = \frac{5}{4}$

R.H.S. of $P(2) = 2 - \frac{1}{2} = \frac{3}{2} = \frac{6}{4}$

$\therefore$ L.H.S. of $P(2) <$ R.H.S. of $P(2)$

$\therefore$ $P(2)$ is true.

Hence, the statement is true for $n > 1\ \forall\ n \in N$.

2. (a) : As $P(n) = (n + 1) + (n + 2) + (n + 3)$

$P(n) = 3n + 6 = 3(n + 2)$

$\therefore$ $P(1) = 3(3) = 9$ which is divisible by 9

$\therefore$ Least value of n is 1.

3. (c) : $P(n) = 5^n - 2^n$

Let $n = 1$ $\therefore$ $P(1) = 3\lambda = 3$

$\therefore$ $\lambda = 1$

Similarly, $n = 5$ $\therefore$ $P(5) = 5^5 - 2^5$

$\Rightarrow P(5) = 3125 - 32 = 3093 = 3 \times 1031$

In this case $\lambda = 1031$

Similarly, we can check the result for other cases and find that the least value of λ and n is 1.

4. (c) : The given expression is

$(1 - 3x + 3x^2 - x^3)^{2n} = (1 - x)^{6n}$

Since the power of the expansion is an even number, therefore there will be only one middle term, which is $\left(\frac{6n}{2}+1\right)^{th}$ term *i.e.* $(3n + 1)^{th}$ term.

$\therefore$ $T_{3n+1} = {}^{6n}C_{3n}(-x)^{3n} = \frac{6n!}{3n!\,3n!}(-x)^{3n}$

5. (c) : $(1 + x + x^2)^n = b_0 + b_1x + b_2x^2 + b_3x^3 + \ldots$
$\ldots + b_{2n}x^{2n}$

Putting $x = 1$, we get

$3^n = b_0 + b_1 + b_2 + b_3 + \ldots + b_{2n}$ ... (i)

Putting $x = -1$, we get

$1 = b_0 - b_1 + b_2 - b_3 + b_4 - \ldots + b_{2n}$ .... (ii)

Adding (i) and (ii), we get

$3^n + 1 = 2(b_0 + b_2 + b_4 + \ldots + b_{2n})$

$\therefore$ $\frac{3^n+1}{2} = b_0 + b_2 + b_4 + b_6 + \ldots + b_{2n}$

6. (a) : The given expression is

$\left(\sqrt[5]{2} + \sqrt[10]{3}\right)^{60} = \left(2^{1/5} + 3^{1/10}\right)^{60}$

Now L.C.M. of 5 and 10 is 10.

$\therefore$ Number of rational terms

$T_{r+1} = {}^{60}C_r(2^{1/5})^{60-r}(3^{1/10})^r$

$= {}^{60}C_r\, 2^{\left(12-\frac{r}{5}\right)} \cdot 3^{\frac{r}{10}}$

As $0 \le r \le 60$

$\therefore$ $r = 0, 10, 20, 30, 40, 50, 60$

$\therefore$ Number of rational terms are 7.

$\therefore$ Number of irrational terms equals to $61 - 7 = 54$.

7. (d) : Here, $T_{r+1} = {}^{10}C_r\,(x\sin^{-1}\alpha)^{10-r}\left(\dfrac{\cos^{-1}\alpha}{x}\right)^r$

$= {}^{10}C_r\, x^{10-2r}\,(\sin^{-1}\alpha)^{10-r}\,(\cos^{-1}\alpha)^r$

For independent term of x, let $10 - 2r = 0$

$\therefore\ r = 5,\ T_6 = {}^{10}C_5(\sin^{-1}\alpha\,\cos^{-1}\alpha)^5$

Here $f(\alpha) = \sin^{-1}\alpha\,\cos^{-1}\alpha$, let $\sin^{-1}\alpha = w$

$\Rightarrow\ f(w) = w\left(\dfrac{\pi}{2} - w\right),\ -\dfrac{\pi}{2} \le w \le \dfrac{\pi}{2}$

$\Rightarrow\ f'(w) = \dfrac{\pi}{2} - 2w.$

Now, $f'(w) = 0$ gives $w = \pi/4$

$\therefore\ f\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^2}{16},\ f\left(-\dfrac{\pi}{2}\right) = -\dfrac{\pi^2}{2},\ f\left(\dfrac{\pi}{2}\right) = 0$

$\therefore\ w \in \left(-\dfrac{\pi^2}{2}, \dfrac{\pi^2}{16}\right)$

$\therefore$ Required range $\left[{}^{10}C_5\left(-\dfrac{\pi^2}{2}\right)^5,\ {}^{10}C_5\left(\dfrac{\pi^2}{16}\right)^5\right]$

$= \left[\dfrac{-{}^{10}C_5\,\pi^{10}}{2^5},\ \dfrac{{}^{10}C_5\,\pi^{10}}{2^{20}}\right]$

8. (d) : Here, $\dfrac{C_1}{C_0} + 2\dfrac{C_2}{C_1} + 3\dfrac{C_3}{C_2} + ... + n\dfrac{C_n}{C_{n-1}}$

$= \dfrac{n}{1} + 2\dfrac{n(n-1)}{2\cdot n} + \dfrac{3n(n-1)(n-2)}{6} \times \dfrac{2}{n(n-1)} + ... + n\cdot\dfrac{1}{n}$

$= n + (n - 1) + (n - 2) + ... + 1$

$= \dfrac{n(n+1)}{2}$ $\left[\because \text{Sum of first } n \text{ natural numbers} = \dfrac{n(n+1)}{2}\right]$

9. (a) : The given expression is $\left(\dfrac{1}{x} + x^{\log_{10} x}\right)^5$

Now, $T_3 = {}^5C_2\left(\dfrac{1}{x}\right)^3 (x^{\log_{10} x})^2$

Now, $10\left(\dfrac{1}{x}\right)^3 \cdot x^{2\log_{10} x} = 1000$

$\Rightarrow\ x^{2\log_{10} x - 3} = 10^2 \ \Rightarrow\ 2\log_{10} x - 3 = \log_x 10^2$

$\Rightarrow\ 2\log_{10} x - 3 = \dfrac{2}{\log_{10} x}$

$\Rightarrow\ 2a - 3 = \dfrac{2}{a}$ [where $a = \log_{10} x$] $\Rightarrow 2a^2 - 3a - 2 = 0$

$\Rightarrow\ 2a(a-2) + 1(a-2) = 0\ \Rightarrow\ (2a+1)(a-2) = 0$

$\therefore\ a = 2$ and $a \ne -\dfrac{1}{2}$ $\quad\therefore\ \log_{10}x = 2\ \Rightarrow\ x = 100$

10. (d) : Let $(t_{r+1})^{\text{th}}$ term is the greatest term. t_{r+1} will be greatest when $\dfrac{t_{r+1}}{t_r} \ge 1$ and $\dfrac{t_{r+2}}{t_{r+1}} \le 1$

Here the given expansion is $(2x - 3y)^{28}$

$\therefore\ t_{r+1} = {}^{28}C_r\,(2x)^{28-r}\cdot(-3y)^r$

$t_r = {}^{28}C_{r-1}\,(2x)^{29-r}\,(-3y)^{r-1}$

$t_{r+2} = {}^{28}C_{r+1}\,(2x)^{27-r}\,(-3y)^{r+1}$

Now, $\dfrac{t_{r+1}}{t_r} \ge 1$ gives $\dfrac{29-r}{r}\cdot\left|\dfrac{-3y}{2x}\right| \ge 1$

$\Rightarrow\ \dfrac{29-r}{r}\cdot\left|\dfrac{-2}{3}\right| \ge 1$ $\quad[\because x = 9 \text{ and } y = 4]$

$\Rightarrow\ 2(29 - r) \ge 3r\ \Rightarrow\ 5r \le 58 \Rightarrow r \le \dfrac{58}{5} = 11\dfrac{3}{5}$

and $\dfrac{t_{r+2}}{t_{r+1}} \le 1$ gives $\dfrac{28-r}{r+1}\left|\dfrac{-3y}{2x}\right| \le 1$

$\Rightarrow\ \dfrac{28-r}{r+1} \times \left|\dfrac{-3\times4}{2\times9}\right| \le 1\ \Rightarrow\ \dfrac{28-r}{r+1}\cdot\dfrac{2}{3} \le 1$

$\Rightarrow\ 56 - 2r \le 3r + 3\ \Rightarrow\ 53 \le 5r$

$\Rightarrow\ r \ge \dfrac{53}{5} = 10\dfrac{3}{5}$

$\therefore\ t_{r+1}$ will be greatest when

$10\dfrac{3}{5} \le r \le 11\dfrac{3}{5}$ *i.e.* when 11

$\therefore\ t_{12}$ is greatest.

11. (b) : We are given that sum of n terms of an A.P. is $cn(n - 1)$.

Let $S_n = cn(n - 1)\ \therefore\ t_n = S_n - S_{n-1}$

$= cn(n - 1) - c(n - 1)(n - 2)$

$= c[(n^2 - n) - (n^2 - 3n + 2)]$

$= c(2n - 2) \Rightarrow t_n = 2c(n - 1)$

$\Rightarrow\ t_n^2 = 4c^2(n^2 - 2n + 1)$

Now, $\sum t_n^2 = 4c^2\sum(n^2 - 2n + 1)$

$= 4c^2 n\left[\dfrac{2n^2 + 3n + 1 - 6n - 6 + 6}{6}\right]$

$= \dfrac{4c^2 n}{6}(2n^2 - 3n + 1) = \dfrac{2}{3}c^2 n(2n-1)(n-1)$

12. (c) : Let the polygon have n sides. Then sum of all the interior angles $= (2n - 4)90°$

$\therefore\ (2n-4)90 = \dfrac{n}{2}[2(120) + (n-1)5]$

$\Rightarrow\ 5n^2 - 125n + 720 = 0$

$\Rightarrow\ (n - 16)(n - 9) = 0 \ \Rightarrow\ n = 16, 9$

When $n = 16$, the largest angle is given by

$120 + (16 - 1)5 = 195$, [$\because t_n = a + (n - 1)d$]

which is impossible and hence $n = 9$.

13. (a) : Let the progression be

1, *, *, *, ..., *, 31

Here $a = 1$, $t_{m+2} = 31$. Let the common difference be d.

$\therefore\ 1+\overline{(m+2-1)}\,d = 31 \Rightarrow (m+1)d = 30 \Rightarrow d = \dfrac{30}{m+1}$

Given that, $\dfrac{7^{\text{th}}\text{ mean}}{(m-1)^{\text{th}}\text{ mean}} = \dfrac{5}{9}$

$$\Rightarrow \frac{1+7d}{1+(m-1)d} = \frac{5}{9} \quad \Rightarrow \frac{1+\frac{7\times 30}{m+1}}{1+\frac{30(m-1)}{m+1}} = \frac{5}{9}$$

$\Rightarrow$ On solving, we get $m = 14$

14. (b) : According to the given problem, we have

$$\frac{a+b}{2} = 2\sqrt{ab} \ \Rightarrow\ a+b = 4\sqrt{ab}$$

$\Rightarrow\ a^2 + 2ab + b^2 = 16ab$

$$\Rightarrow a^2 - 14ab + b^2 = 0 \Rightarrow \left(\frac{a}{b}\right)^2 - 14\left(\frac{a}{b}\right) + 1 = 0$$

$$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}, \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

$\therefore$ The required ratio is $2+\sqrt{3} : 2-\sqrt{3}$.

15. (d) : $P = \dfrac{3}{17} + \dfrac{33}{17^2} + \dfrac{333}{17^3} + \ldots.\infty$ ... (i)

$\dfrac{P}{17} = \dfrac{3}{17^2} + \dfrac{33}{17^3} + \ldots.\infty$ ... (ii)

(i) – (ii) gives,

$$P\left(1-\frac{1}{17}\right) = \frac{3}{17} + \frac{30}{17^2} + \frac{300}{17^3} + \ldots. + \infty$$

$$\Rightarrow \frac{16}{17}P = \frac{3}{17}\left(1 + \frac{10}{17} + \frac{100}{17^2} + \ldots. + \infty\right)$$

$$\Rightarrow \frac{16}{17}P = \frac{3}{7} \quad \Rightarrow \quad P = \frac{51}{112}$$

16. (a) : Let the progression be

2, *, *, *, ..., *, 20

Let the common difference be d.

$\therefore\ t_{10} = 20 \Rightarrow 2 + 9d = 20 \Rightarrow d = 2$

First mean $= a + d = 2 + 2 = 4$

Second mean $= a + 2d = 2 + 4 = 6$

Third mean $= a + 3d = 2 + 6 = 8$

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

Eight mean $= a + 8d = 2 + 16 = 18$

$\therefore$ Sum of all the 8 means $= \dfrac{8}{2}[4+18] = 4[22] = 88$

17. (d) : Let the roots be $\dfrac{x}{y}, x, xy$.

$$\text{Product of the roots} = x^3 = -\frac{d}{a} \Rightarrow x = \left(\frac{-d}{a}\right)^{1/3}$$

Now putting $x = \left(-\dfrac{d}{a}\right)^{1/3}$ in the given equation as x is its root, we get

$$a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{\frac{2}{3}} + c\left(-\frac{d}{a}\right)^{\frac{1}{3}} + d = 0$$

$$\Rightarrow b\left(\frac{d}{a}\right)^{\frac{2}{3}} = c\left(\frac{d}{a}\right)^{\frac{1}{3}} \Rightarrow b^3\left(\frac{d}{a}\right)^2 = c^3\left(\frac{d}{a}\right)$$

$$\Rightarrow b^3\left(\frac{d}{a}\right) = c^3 \left[\text{dividing both sides by } \left(\frac{d}{a}\right)\right]$$

$\Rightarrow\ b^3d = ac^3$

18. (b) : Let the father's age be F and that of his elder and younger child respectively be E and Y.

$\therefore\ F = 2E + 4Y$ ...(i)

$\sqrt{EY} = 4\sqrt{3}$ ...(ii) and $\dfrac{2EY}{E+Y} = 6$ ...(iii)

Solving (ii) and (iii) we get $E = 12$ and $Y = 4$

$\therefore$ (i) becomes

$F = 2 \times 12 + 4 \times 4 = 24 + 16 = 40$

Thus, the father's age is 40 years.

19. (d) : $a_1 = 2,\ a_n - a_{n-1} = 2n\ \forall\ n \ge 2$

$a_2 = a_1 + 2\cdot 2 \Rightarrow a_2 = 6$

$a_3 = a_2 + 3\cdot 2 \Rightarrow a_3 = 12,\ a_4 = 20$

$S = 2 + 6 + 12 + 20 + \ldots.. + a_n$ ....(i)

$S = 2 + 6 + 12 + \ldots. + a_{n-1} + a_n$ ....(ii)

(i) – (ii) gives

$2 + 4 + 6 + 8 + \ldots. + n$ terms $- a_n = 0$

$\Rightarrow\ a_n = 2\ (1 + 2 + 3 + \ldots.. +$ to n terms)

$a_n = 2\dfrac{n(n+1)}{2} = n(n+1) \quad \therefore \quad \Sigma a_n = \Sigma n^2 + \Sigma n$

$\Rightarrow\ S_n = \dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2} \quad \therefore \quad S_{20} = 3080$

20. (c) : Let the sides of triangle be a, ar, ar^2.

$\therefore\ r > 1\ \ \therefore\ ar^2$ is greatest side $\therefore\ a + ar > ar^2$

$\Rightarrow\ r^2 - r - 1 = 0$

$$\Rightarrow \frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2} \Rightarrow 1 < r < \frac{1+\sqrt{5}}{2} \quad (\because r > 1)$$

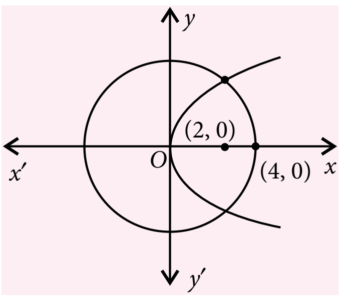
∴ $[r]=1$. Also $-\frac{1+\sqrt{5}}{2}<-r<-1$ (∵ $[-r]=2$)

∴ $[r]+[-r]=1-2=-1$

21. (c) : For parabola $y^2=4ax$, the line $y=mx+c$ will be tangent if $c=\frac{a}{m}$.

Equation of tangent to parabola $y^2=8x$ is $y=mx+\frac{2}{m}$.

Now line to be chord of the circle $x^2+y^2=16$ if distance from (0, 0) to the line will be less than the radius of circle.



$$\therefore\ \frac{\frac{2}{m}}{\sqrt{1+m^2}}<4$$

$$\Rightarrow\ \frac{2}{m\sqrt{1+m^2}}<4$$

$\Rightarrow\ 1<4m^2(1+m^2)$

$\Rightarrow\ 4m^4+4m^2-1>0$

$$\Rightarrow\ m^4+m^2-\frac{1}{4}>0\ \Rightarrow\ \left(m^2+\frac{1}{2}\right)^2-\frac{1}{2}>0$$

$$\Rightarrow\ \left(m^2+\frac{1}{2}+\frac{1}{\sqrt{2}}\right)\left(m^2+\frac{1}{2}-\frac{1}{\sqrt{2}}\right)=0$$

$$\Rightarrow\ m^2>-\frac{1}{2}+\frac{1}{\sqrt{2}}\text{ or }m^2>\frac{\sqrt{2}-1}{2}$$

$$\Rightarrow\ |m|>\sqrt{\frac{\sqrt{2}-1}{2}}$$

$$\Rightarrow\ m\in\left(-\infty,-\sqrt{\frac{\sqrt{2}-1}{2}}\right)\cup\left(\sqrt{\frac{\sqrt{2}-1}{2}},\infty\right)$$

22. (d) : Equation of tangent at $(t^2, 2t)$ is

$x=ty-t^2$ ...(i)

Equation of normal to ellipse at $(\sqrt{5}\cos\alpha, 2\sin\alpha)$

is $\sqrt{5}x\sec\alpha-2y\,\mathrm{cosec}\,\alpha=1$ ...(ii)

Given (i) = (ii)

$$\Rightarrow\ \sqrt{5}\sec\alpha=\frac{2\,\mathrm{cosec}\,\alpha}{t}=-\frac{1}{t^2}$$

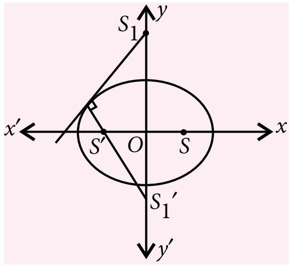
$\Rightarrow\ \cos\alpha=-\sqrt{5}t^2$ and $\sin\alpha=-2t$

$\Rightarrow\ \cos^2\alpha+\sin^2\alpha=5t^4+4t^2=1$

$$\Rightarrow\ t^2=\frac{1}{5}.\ \text{Now, } \frac{\sin\alpha}{\cos\alpha}=\frac{2t}{-\sqrt{5}t^2}=\frac{2}{\sqrt{5}}\times\frac{1}{t}=\frac{2}{\sqrt{5}}\times(\pm 5)$$

∴ $\tan\alpha=\pm 2\Rightarrow\tan\alpha=-2,\ \tan\alpha=2$

23. (c) : Given $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$$\therefore\ OS=ae=a\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{a^2-b^2}$$

So two points on the minor axis are

$S_1(0,\sqrt{a^2-b^2}),\ S_1'(0,-\sqrt{a^2-b^2})$

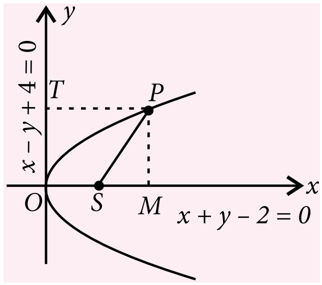
Let tangent to the ellipse be $y=mx+c=mx+\sqrt{a^2m^2+b^2}$ where m is parameter.

Now sum of the squares of perpendiculars on the tangent from the points S_1 and S'_1 is

$$\left(\frac{\sqrt{a^2-b^2}-\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right)^2+\left(-\frac{\sqrt{a^2-b^2}-\sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}\right)^2$$

$$=2\left(\frac{a^2-b^2+a^2m^2+b^2}{1+m^2}\right)=\frac{2a^2(1+m^2)}{1+m^2}=2a^2$$

24. (c) : Let $P(x, y)$ be any point on the parabola and say PM and PT are perpendicular from P on the axis and tangent at the vertex.

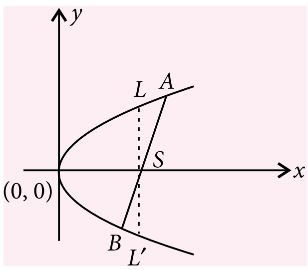


∴ $(PM)^2=\text{latus rectum}\ (PT)$

$$\therefore\ \left(\frac{x+y-2}{\sqrt{1^2+1^2}}\right)^2=2\left(\frac{x-y+4}{\sqrt{1^2+1^2}}\right)$$

$$\Rightarrow\ (x+y-2)^2=2\sqrt{2}(x-y+4)$$

25. (c) : We know that semi latus rectum of a parabola is the H.M. between the segment of any focal chord of a parabola.



∴ AS, 4, SB are in H.P.

$$\Rightarrow\ 4=2\times\frac{AS\cdot SB}{AS+SB}$$

$$\Rightarrow\ 4=\frac{2\times 4SB}{4+SB}\ \Rightarrow\ SB=4$$

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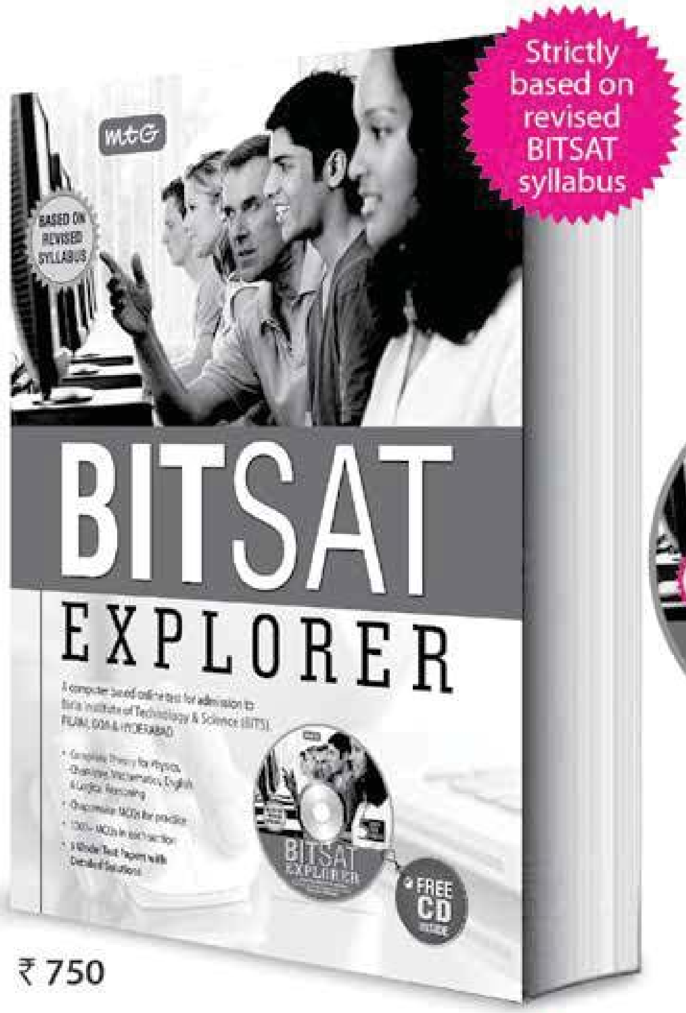

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26. (a) : $y = 4x^2 \Rightarrow x^2 = \frac{1}{4}y$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{16} = 1$ becomes $\frac{y}{4a^2} - \frac{y^2}{16} = 1$

$\Rightarrow 4y - a^2y^2 = 16a^2 \Rightarrow a^2y^2 - 4y + 16a^2 = 0$

$\Rightarrow D \geq 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2(16a^2) \geq 0 \Rightarrow 1 - 4a^4 \geq 0$

$\Rightarrow (\sqrt{2}a^2)^2 \leq 1 \Rightarrow |\sqrt{2}a| \leq 1 \Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

27. (b) : Equation of normal to the curve $y^2 = 4x$ at $(m^2, 2m)$ is taken as

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(m^2, 2m)}}(x - x_1)$$

$\Rightarrow (y - 2m) = -m(x - m^2)$

$\Rightarrow y + mx - 2m - m^3 = 0$ ...(i)

Similarly normal to $y^2 - 2x + 6 = 0$ at $\left(\frac{1}{2}t^2 + 3, t\right)$ is

$y + t(x - 3) - t - \frac{1}{2}t^3 = 0$ ...(ii)

Shortest distance between two curves exist along the common normal.

Let (i) and (ii) are same

$\therefore -2m - m^3 = -4m - \frac{1}{2}m^3 \Rightarrow m = 0, m = \pm 2$

Points on the parabolas $(m^2, 2m) = (4, 4)$

and $\left(\frac{1}{2}m^2 + 3, m\right) = (5, 2)$

$\therefore$ Shortest distance $= \sqrt{(5-4)^2 + (4-2)^2} = \sqrt{5}$

28. (a) : Equation of chord of contact of tangent from (x_1, y_1) to the ellipse is

$\frac{xx_1}{52} + \frac{yy_1}{13} = 0 \Rightarrow m_1 = \text{slope} = \frac{-x_1}{4y_1}$

Again chord of contact of tangent from (x_2, y_2) is

$\frac{xx_2}{52} + \frac{yy_2}{13} = 0 \therefore m_2 = -\frac{x_2}{4y_2}$

Given tangents are right angle $\therefore m_1m_2 = -1$

$\Rightarrow \frac{x_1}{4y_1} \cdot \frac{x_2}{4y_2} = -1 \Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{16}{1}$

29. (c) : Any point on the ellipse

$\frac{x^2}{(3\sqrt{2})^2} + \frac{y^2}{(4\sqrt{2})^2} = 1$ can be taken as

$(3\sqrt{2}\cos\theta, 4\sqrt{2}\sin\theta)$ and slope of the tangent

$-\frac{b^2x}{a^2y} = -\frac{32(3\sqrt{2}\cos\theta)}{18(4\sqrt{2}\sin\theta)} = -\frac{4}{3}\cot\theta$ ....(i)

Given slope of the tangent $= -\frac{4}{3}$ ....(ii)

From (i) and (ii), we get $\cot\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

Hence equation of tangent is

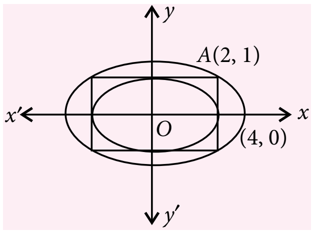
$\frac{x \cdot \frac{1}{\sqrt{2}}}{3\sqrt{2}} + \frac{y \cdot \frac{1}{\sqrt{2}}}{4\sqrt{2}} = 1 \Rightarrow \frac{x}{6} + \frac{y}{8} = 1$

Hence A (6, 0), B (0, 8)

Area of $\Delta OAB = \frac{1}{2} \times 6 \times 8 = 24$ sq. units

30. (a) : The given ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$

i.e., the point A, the corner of the rectangle in 1st quadrant is (2, 1). Again the ellipse circumscribing the rectangle passes through the point (4, 0), so its equation is $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$.



$A(2, 1)$ lies on the above ellipse

$\Rightarrow \frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3}$

Thus the equation to the required ellipse is

$\frac{x^2}{16} + \frac{3}{4}y^2 = 1 \Rightarrow x^2 + 12y^2 = 16$

Solution Sender of Maths Musing

SET-176

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SET-177

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MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The readers' & comments and suggestions regarding the problems and solutions offered are always welcome.

1. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals

(a) 0 (b) 1 (c) $\frac{1}{mn}$ (d) $\frac{1}{m}+\frac{1}{n}$

2. If $\vec{a} = 2\vec{i} - 2\vec{j} + \vec{k}$ then
$[\vec{i}\ \vec{a}\ \vec{a}\times\vec{i}] + [\vec{j}\ \vec{a}\ \vec{a}\times\vec{j}] + [\vec{k}\ \vec{a}\ \vec{a}\times\vec{k}] =$

(a) –9 (b) 9 (c) 18 (d) –18

3. Let $S_1 = \sum_{j=1}^{10} j(j-1)(^{10}C_j), S_2 = \sum_{j=1}^{10} j\,(^{10}C_j)$

and $S_3 = \sum_{j=1}^{10} j^2\,(^{10}C_j)$

Statement –I: $S_3 = 55 \times 2^9$

Statement –II: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

(a) Both Statement –I and Statement –II are true and Statement –II is correct explanation of Statement –I.

(b) Both Statement –I and Statement –II are true and Statement –II is not correct explanation of Statement –I.

(c) Statement –I is true but Statement –II is false.

(d) Statement –I is false but Statement –II is true.

4. Let $f : [0, \sqrt{3}] \to \left[0, \frac{\pi}{3} + \log_e 2\right]$ defined by $f(x) = \log_e \sqrt{x^2+1} + \tan^{-1} x$, then $f(x)$ is

(a) one – one and onto

(b) one – one but not onto

(c) onto but not one – one

(d) neither one – one nor onto

5. In $\Delta PQR, \angle R = \frac{\pi}{2}$, If $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are roots of the equation $ax^2 + bx + c = 0$, $(a \neq 0)$ then

(a) $a + b = c$ (b) $b + c = a$

(c) $a + c = b$ (d) $b = c$

6. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1+|\sin\theta|)z_1 + (|\cos\theta|-1)z_2 - \sqrt{2}z_3 = 0$. If at least one of the complex numbers z_1, z_2, z_3 is non- zero then number of possible values of θ is

(a) ∞ (b) 4 (c) 2 (d) 8

7. $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx =$

(a) 0 (b) 5 (c) 1 (d) 4

8. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants, is of

(a) first order and second degree

(b) first order and first degree

(c) second order and first degree

(d) second order and second degree

9. The number of critical points of the curve $f(x) = (x - 2)^{2/3}(2x + 1)$ are

(a) 1 (b) 2 (c) 3 (d) 0

10. The sides of a triangle are $3x + 4y$, $4x + 3y$ and $5x + 5y$ where $x, y > 0$ then the triangle is

(a) right angled (b) obtuse angled

(c) equilateral (d) isosceles

SOLUTIONS

1. (a) : $T_m = 1/n = a + (m-1)d$ ...(i)

$T_n = 1/m = a + (n-1)d$ ...(ii)

From (i) and (ii) we get, $a = \frac{1}{mn}, d = \frac{1}{mn} \Rightarrow a - d = 0$

2. (d) : $-\left\{(\vec{a}\times\vec{i})^2 + (\vec{a}\times\vec{j})^2 + (\vec{a}\times\vec{k})^2\right\} = -2|\vec{a}|^2$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

3. (c) : $S_1 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)(j-2)!(10-j)!}$

$= 90\sum_{j=2}^{10}\frac{8!}{(j-2)!(8-(j-2))!} = 90\cdot 2^8$

$S_2 = \sum_{j=1}^{10} j\frac{10!}{j(j-1)!(9-(j-1))!}$

$= 10\sum_{j=1}^{10}\frac{9!}{(j-1)!(9-(j-1))!} = 10\cdot 2^9$

$S_3 = \sum_{j=1}^{10}[j(j-1)+j]\frac{10!}{j!(10-j)!}$

$= \sum_{j=1}^{10} j(j-1)^{10}C_j + \sum_{j=1}^{10} j\ {}^{10}C_j = 90\cdot 2^8 + 10\cdot 2^9$

$= 90\cdot 2^8 + 20\cdot 2^8 = 110\cdot 2^8 = 55\cdot 2^9$

4. (a) : $f'(x) = \frac{x+1}{x^2+1} \Rightarrow f(x)$ is increasing in $[0,\sqrt{3}]$

5. (a) : $\frac{P}{2}+\frac{Q}{2}=\frac{\pi}{4} \Rightarrow c = a+b$

6. (b) : If z_1, z_2, z_3 are collinear and $az_1 + bz_2 + cz_3 = 0$ then $a + b + c = 0$ .Hence $1+|\sin\theta|+|\cos\theta|-1-\sqrt{2}=0$

$\Rightarrow |\sin\theta|+|\cos\theta| = \sqrt{2} \Rightarrow \theta = \frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

7. (a) : Put $x - 3 = t$

8. (c) : Differentiate the given equations for 2 times and eliminate A, B

9. (b) : $f'(x) = 0$ or undefined at $x = 1, 2$

10. (b) : Let

$\cos C = \frac{(3x+4y)^2+(4x+3y)^2-(5x+5y)^2}{2(3x+4y)(4x+3y)} < 0$

Computer science losing out, mechanical hot pick in BTech

Versatile Job Opening Key To New Trend

Engineering is being revisited. Even as seats in this professional course are reducing across the country (from 16.3 lakh in 2013-14 to about 14.7 lakh this year), mechanical engineering seems to be emerging as the hottest pick in times of uncertainty in the IT and software industry.

While engineering continues to be a big draw, its 70-odd options undergo a life cycle of their own. Experts say industry growth, which translates into more jobs and higher incomes, is what decides the path that colleges and students take. And many feel the sun is setting on the computer science engineering stream. While 25.44% of all students opted for it in 2013-14, that has dropped to around 24% this year.

At the same time, mechanical engineering is racing ahead after pipping electronics and communications, which used to be the second most popular choice for four years. While 20.22% of students chose mechanical engineering in the past four years, that proportion has increased to 21.6% now.

Core courses like civil and electrical engineering are also expected to be top on the charts.

COURSE CORRECTION

STREAM	INTAKE				
	2013/14	2014/15	2015/16	2016/17	2017/18
Computer Science	4,14,762	4,08,559	3,88,077	3,68,267	3,53,825
Mechanical	3,30,331	3,58,052	3,47,480	3,35,603	3,17,444
Electronics and Comm	3,43,180	3,41,916	3,22,292	3,02,800	2,80,653
Civil	2,22,510	2,55,139	2,46,903	2,40,168	2,28,841
Electrical	2,10,803	2,18,071	2,09,653	2,00,570	1,90,780

Although data from the All India Council for Technical Education (AICTE) shows that seats in undergraduate engineering are reducing, experts feel the course will continue to have lakhs of takers. "Engineering has become a broad-based course like BA, BCom and BSc. From here, students go on to do several courses, says IIT-Madras director Bhaskar Ramamurthy.

"Mechanical engineering is a great branch. One can fit into a lot of industries after ME," adds Ramamurthy. " But given some amount of uncertainty in the IT sector, there are more takers for mechanical-because mechanical students can join IT companies, but the reverse is not possible."

From information technology to leather technology, each of these courses tests the ability of individuals to engineer a lasting impression, may be an innovation, a new process, a fresh thought. But generalized courses are likely to be the flavour for the coming few years as several industries see a departure from the old.

G D Yadav, vice-chancellor of the Institute of Chemical Technology, says mechanical is rising in popularity because manufacturing industry needs these engineers. "There is so much new construction, new infrastructure, machinery and mechanical engineers are needed everywhere." Hence, computer science and mechanical engineers will be an equal force in the coming year," he adds.

If the current trend in the USA in anything to go by, the demand for chemical engineers too is likely to rise. Enrolment in this programme has doubled in American universities this year thanks to shale gas companies that have upped their recruitment numbers, says Yadav. Renewable energy and electric cars, many forecast, will also see the energy engineering space powering up.

Courtesy: TOI

JEE (ADVANCED)

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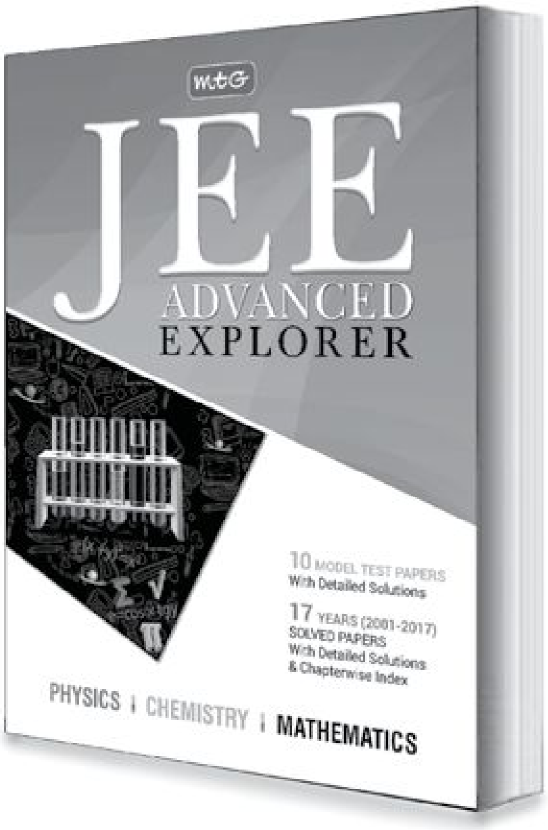

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MATHS MUSING

SOLUTION SET-177

1. (c) : $x^2 - (y + 42)^2 = 2012 - (42)^2 = 248 = 124 \times 2$

$\therefore\ x + y + 42 = 124,\ x - y - 42 = 2$

$\Rightarrow\ x = 63,\ y = 19,\ x - 3y = 6$

2. (c) : Given $g(x) = \dfrac{(x-1)^n}{\log\cos^m(x-1)}, 0 < x < 2,$

$m \neq 0$, n are integers and $|x-1| = \begin{cases} x-1; x \geq 1 \\ 1-x; x < 1 \end{cases}$

The left hand derivative of $|x-1|$ at $x = 1$ is $p = -1$

Also, $\lim\limits_{x \to 1^+} g(x) = p = -1$

$$\Rightarrow \lim_{h \to 0} \frac{(1+h-1)^n}{\log\cos^m(1+h-1)} = -1$$

$$\Rightarrow \lim_{h \to 0} \frac{h^n}{\log\cos^m h} = -1$$

$$\Rightarrow \lim_{h \to 0} \frac{n \cdot h^{n-1}}{m\dfrac{1}{\cos h}(-\sin h)} = -1 \quad \text{[Using L'Hospital's rule]}$$

$$\Rightarrow \left(\frac{n}{m}\right)\lim_{h \to 0} \frac{h^{n-2}}{\left(\dfrac{\tan h}{h}\right)} = 1 \ \Rightarrow\ n = 2 \text{ and } \frac{n}{m} = 1$$

$\Rightarrow\ m = n = 2$

3. (d) : We have,

$$\frac{dx}{dt} = a\left[-\sin t\sqrt{\cos 2t} - \frac{\cos t \sin 2t}{\sqrt{\cos 2t}}\right] = \frac{-a\sin 3t}{\sqrt{\cos 2t}}$$

and $$\frac{dy}{dt} = a\left[\cos t\sqrt{\cos 2t} - \frac{\sin t \sin 2t}{\sqrt{\cos 2t}}\right] = \frac{a\cos 3t}{\sqrt{\cos 2t}}$$

$$\therefore\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\cot 3t \ \Rightarrow\ \frac{d^2y}{dx^2} = 3\,\text{cosec}^2 3t \cdot \frac{dt}{dx}$$

$$= \frac{-3\,\text{cosec}^2 3t \cdot \sqrt{\cos 2t}}{a\sin 3t} = -\left(\frac{3}{a}\right)\text{cosec}^3 3t\sqrt{\cos 2t}$$

$$\therefore\ \frac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{(1+\cot^2 3t)^{3/2}}{\left(-\dfrac{3}{a}\right)\text{cosec}^3 3t\sqrt{\cos 2t}}$$

$$\Rightarrow \left|\frac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2}\right|_{t=\pi/6} = \frac{\sqrt{2}a}{3}$$

4. (d) : We have, $\displaystyle\int \frac{\sin^3 x/2}{\cos\frac{x}{2}\sqrt{\cos^3 x + \cos^2 x + \cos x}}dx$

$$= \frac{1}{2}\int \frac{2\sin(x/2)\cos(x/2)(2\sin^2 x/2)}{2\cos^2\frac{x}{2}\sqrt{\cos^3 x + \cos^2 x + \cos x}}dx$$

$$= \frac{1}{2}\int \frac{\sin x(1-\cos x)}{(1+\cos x)\sqrt{\cos^3 x + \cos^2 x + \cos x}}dx$$

$$= -\frac{1}{2}\int \frac{1-t}{(1+t)\sqrt{t^3+t^2+t}}dt,$$

where $t = \cos x$ and $-\sin x dx = dt$

$$= \frac{1}{2}\int \frac{t^2-1}{(t+1)^2\sqrt{t^3+t^2+t}}dt$$

$$= \frac{1}{2}\int \frac{t^2-1}{(t^2+2t+1)\sqrt{t^3+t^2+t}}dt$$

$$= \frac{1}{2}\int \frac{1-\dfrac{1}{t^2}}{\left(t+2+\dfrac{1}{t}\right)\sqrt{t+\dfrac{1}{t}+1}}dt. = \int \frac{1}{u^2+1}du$$

where $t + \dfrac{1}{t} + 1 = u^2 \Rightarrow \left(1 - \dfrac{1}{t^2}\right)dt = 2udu$

$= \tan^{-1} u + c$

$$= \tan^{-1}\sqrt{t+\frac{1}{t}+1} + c = \tan^{-1}\sqrt{\cos x + \sec x + 1} + c$$

5. (d) : The equation representing the coaxial system of circles is $x^2 + y^2 + 2gx + c + \lambda(x^2 + y^2 + 2fy + k) = 0$

$$\Rightarrow\ x^2 + y^2 + \frac{2g}{1+\lambda}x + \frac{2f\lambda}{1+\lambda}y + \frac{c+k\lambda}{1+\lambda} = 0 \quad \ldots\text{(i)}$$

The coordinates of the centre of this circle are

$$\left(-\frac{g}{1+\lambda}, -\frac{f\lambda}{1+\lambda}\right) \quad \ldots\text{(ii)}$$

and radius $= \sqrt{\dfrac{g^2+f^2\lambda^2-(c+k\lambda)(1+\lambda)}{(1+\lambda)^2}}$

For the limiting points, we must have radius = 0

$\Rightarrow\ g^2 + f^2\lambda^2 - (c + k\lambda)(1 + \lambda) = 0$

$\Rightarrow\ \lambda^2(f^2 - k) - \lambda(c + k) + (g^2 - c) = 0 \quad \ldots\text{(iii)}$

MPP-6 CLASS XII ANSWER KEY

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(b)
6.	(a)	**7.**	(b,c,d)	**8.**	(a)	**9.**	(a,b,c)	**10.**	(a)
11.	(a,b,c,d)	**12.**	(a,d)	**13.**	(a,c,d)	**14.**	(c)	**15.**	(c)
16.	(c)	**17.**	(1)	**18.**	(2)	**19.**	(4)	**20.**	(0)

Let λ_1 and λ_2 be the roots of this equation. Then,

$$\lambda_1+\lambda_2=\frac{c+k}{f^2-k} \text{ and } \lambda_1\lambda_2=\frac{g^2-c}{f^2-k} \quad \ldots\text{(iv)}$$

Thus, the coordinates of limiting points L_1 and L_2 are

$$L_1\left(-\frac{g}{1+\lambda_1},\frac{-f\lambda_1}{1+\lambda_1}\right) \text{ and } L_2\left(\frac{-g}{1+\lambda_2},\frac{-f\lambda_2}{1+\lambda_2}\right)$$

[From eq. (iv)]

Now, L_1L_2 will subtend a right angle at the origin, if Slope of OL_1 × Slope of $OL_2 = -1$

$$\Rightarrow \frac{f\lambda_1}{g}\times\frac{f\lambda_2}{g}=-1 \Rightarrow f^2\lambda_1\lambda_2=-g^2$$

$$\Rightarrow f^2\left(\frac{g^2-c}{f^2-k}\right)=-g^2 \Rightarrow f^2(g^2-c)+g^2(f^2-k)=0$$

$$\Rightarrow \frac{c}{g^2}+\frac{k}{f^2}=2$$

6. (a) : Possibilities are (3, 1, 1), (2, 2, 1)

$$\text{Number of ways}=\frac{{}^5C_2\cdot{}^3C_2\cdot{}^1C_1}{2!}+\frac{{}^5C_3\cdot{}^2C_1\cdot{}^1C_1}{2!}$$

= (10 + 15) = 25

7. (c) : We have, $f(x)=g_9(x)=\dfrac{9^x}{9^x+3}$ ...(i)

$$\text{Now, } \sum_{r=1}^{1995} f\left(\frac{r}{1996}\right)=f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+f\left(\frac{3}{1996}\right)$$

$$+\ldots+f\left(\frac{1993}{1996}\right)+f\left(\frac{1994}{1996}\right)+f\left(\frac{1995}{1996}\right)$$

$$=\left\{f\left(\frac{1}{1996}\right)+f\left(\frac{1995}{1996}\right)\right\}+\left\{f\left(\frac{2}{1996}\right)+f\left(\frac{1994}{1996}\right)\right\}$$

$$+\left\{f\left(\frac{3}{1996}\right)+f\left(\frac{1993}{1996}\right)\right\}+\ldots+\left\{f\left(\frac{997}{1996}\right)+f\left(\frac{999}{1996}\right)\right\}$$

$$+f\left(\frac{998}{1996}\right)$$

$$=\underbrace{1+1+1\ldots+1+1}_{997\text{ times}}+f\left(\frac{1}{2}\right)=997+\frac{1}{2}\,[\text{from (i)}]=997.5$$

$$\therefore \left[\sum_{r=1}^{1995} f\left(\frac{r}{1996}\right)\right]=997$$

8. (c) : $\displaystyle\sum_{r=0}^{2n} f\left(\frac{r}{2n+1}\right)=f(0)+\sum_{r=1}^{2n} f\left(\frac{r}{2n+1}\right)$

$$=\frac{1}{1+\sqrt{a}}+n=\frac{1}{1+\sqrt{a}}+987 \quad \text{(given)}$$

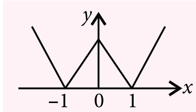
$\therefore$ n = 987

9. (8) : $2^{2003}=2^3\cdot(2^4)^{500}=8(17-1)^{500}=8(17\lambda+1),\lambda\in I$

$\therefore$ Remainder = 8

10. (d) : P. $\displaystyle\lim_{x\to0}\left(\frac{\tan x}{x}\right)^{1/x}=\exp\lim_{x\to0}\frac{\tan x-x}{x^2}=e^0=1$

Q. $f(x)$ is not differentiable at x = 0, ±1



R. Setting $t=\dfrac{1}{u}$ in the second integral,

$$\int_{1/e}^{\tan x}\frac{t\,dt}{1+t^2}+\int_{\tan x}^{e}\frac{u\,du}{1+u^2}=\int_{1/e}^{e}\frac{t\,dt}{1+t^2}=\ln\sqrt{1+t^2}\Big]_{1/e}^{e}=\ln e=1$$

S. $(x-h)^2+(y-k)^2=r^2$, Differentiating twice,

$(x-h)+(y-k)y_1=0,\ 1+(y-k)y_2+y_1^2=0$

$$\therefore\ y-k=-\frac{(1+y_1^2)}{y_2},\ x-h=\frac{y_1}{y_2}(1+y_1^2)$$

$$\therefore\ y_1^2(1+y_1^2)^2+(1+y_1^2)^2=r^2y_2^2$$

or $r^2y_2^2=(1+y_1^2)^3$ $\therefore$ Degree is 2.

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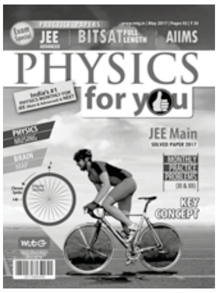

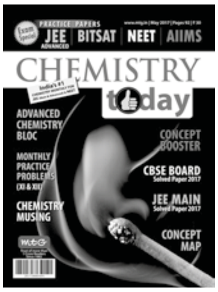

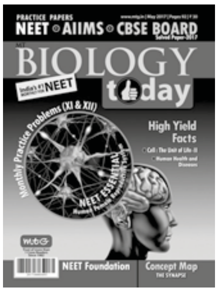

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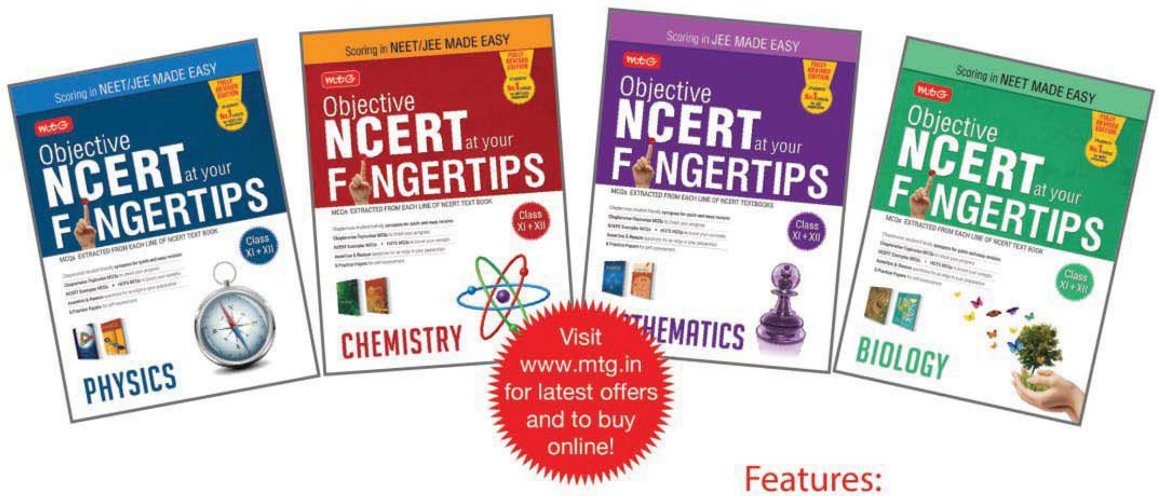

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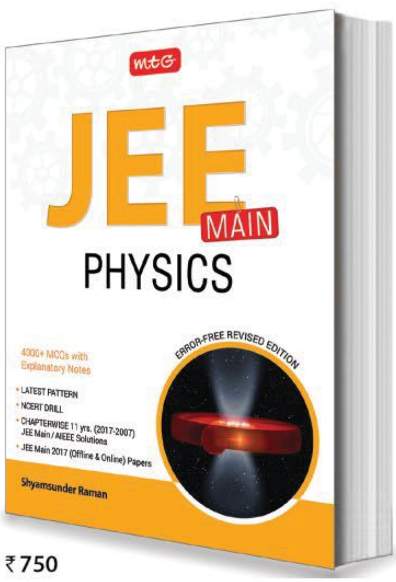

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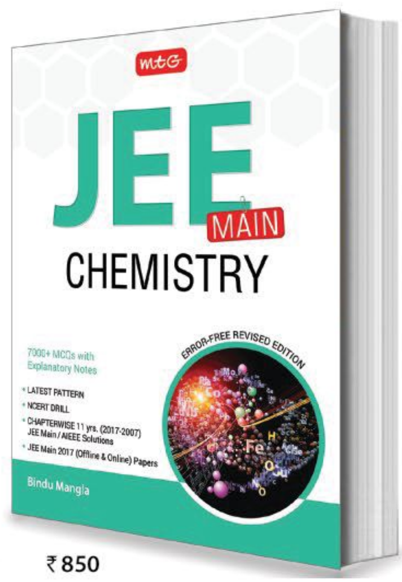

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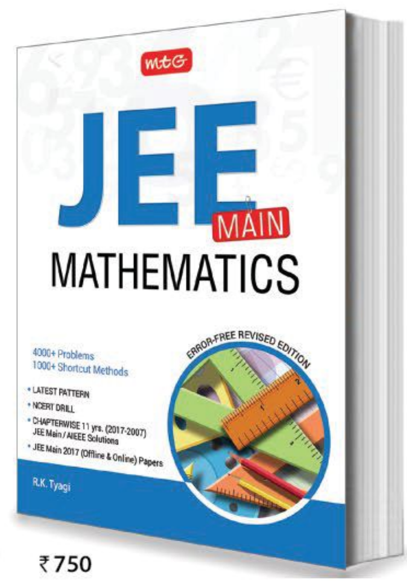

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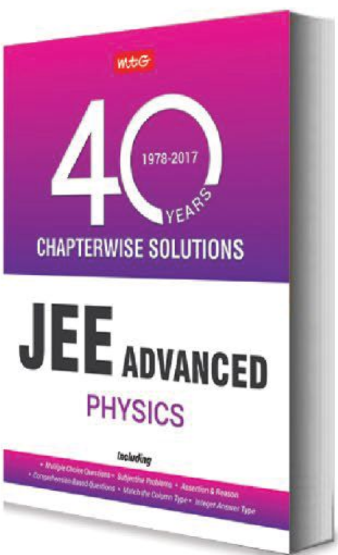

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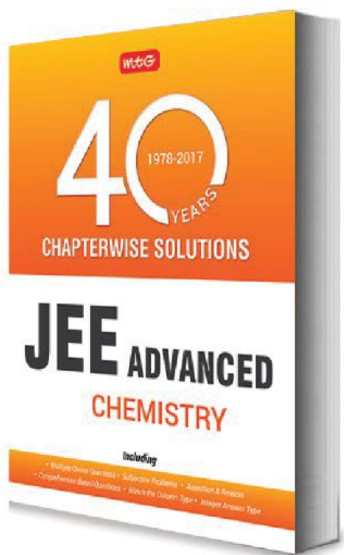

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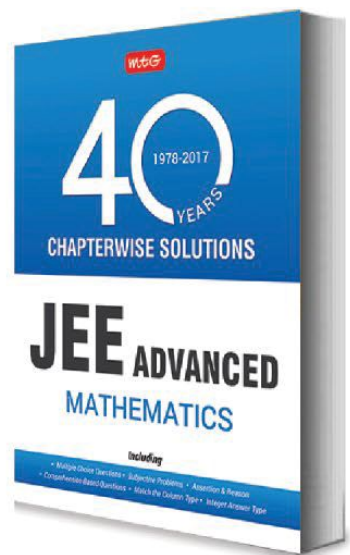

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